Time allowed – 55 minutes (start 12:05 end 13:00)
Total number of questions – 3
Total number of marks – 32
Answer ALL questions
The questions are NOT of equal value
This examination paper has 3 pages.

This paper may be retained by the candidate
Portable battery-powered electronic calculators (without alphabetic keyboards) may be used.
All answers must be in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
The following information is supplied as an aid to memory.

Planck’s constant \( h = 6.626 \times 10^{-34} \) Js

Fundamental charge unit \( e = 1.60 \times 10^{-19} \) C

Speed of light (vacuum) \( c = 3.0 \times 10^8 \) m/s

Electron mass \( = 9.1 \times 10^{-31} \) kg

Neutron mass \( = 1.675 \times 10^{-27} \) kg

Proton mass \( = 1.672 \times 10^{-27} \) kg

Boltzmann’s constant \( k = 1.38 \times 10^{-23} \) JK\(^{-1}\)

Angstrom (Å) = \( 1.0 \times 10^{-10} \) m

Permittivity constant \( \varepsilon_0 = 8.85 \times 10^{-12} \) Fm\(^{-1}\)

Gravitational constant \( G = 6.67 \times 10^{-11} \) Nm\(^2\)/kg\(^2\)

Time-independent Schrödinger Equation: \[-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)\]

Time-dependent Schrödinger Equation: \[-\frac{\hbar^2}{2m} \frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar \frac{\partial\psi(x,t)}{\partial t}\]

\[\int \sin^2(bx)\,dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}\]

\[\int x\sin^2(x)\,dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}\]

\[\int x^2\sin^2(bx)\,dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^2}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}\]

\[\int e^{-bx^2}\,dx = \sqrt{\frac{\pi}{b}}\]

\[\int_0^\infty x^n e^{-bx}\,dx = \frac{n!}{b^{n+1}}\]

\[\sin(2\theta) = 2\sin\theta\cos\theta\]

Bragg’s law: \( n\lambda = 2d\sin\theta \)

Compton Shift: \( \Delta\lambda = \frac{\hbar}{mc}(1 - \cos\theta) \)

Relativistic \( E-p \) relation: \( E^2 = (pc)^2 + (mc^2)^2 \) = Kinetic energy + Rest mass energy

Non-relativistic \( E-p \) relation: \( E = p^2/2m \)
Question 1 [14 Marks]

(a) In one or two sentences, explain briefly the Photoelectric Effect.

(b) Light of frequency $\nu$ and relatively low intensity $I$ shines on a metal cathode and the photoelectrons are collected by an anode at a positive potential $V$. The photocurrent $i$ is reduced to zero by a reverse potential $V = -V_0$ (where $V_0 > 0$). The work function of the cathode is $\phi$. Assuming that the quantum theory of light is correct, sketch 4 graphs showing the variation of:

(i) Photocurrent $i$ with light intensity $I$ for $V > 0$ and $\nu > \nu_0$, the cutoff frequency.
(ii) Photocurrent $i$ with frequency $\nu$ for $V > 0$ and constant $I$.
(iii) Stopping potential $V_0$ with intensity $I$ at constant $\nu$.
(iv) Stopping potential $V_0$ with frequency $\nu$ at constant $I$.

You should identify key features (intercepts, gradients, etc. where possible).

(c) What are the three aspects of the photoelectric effect that cannot be explained by classical physics? How did Einstein’s explanation of this effect overcome the problems with the classical interpretation?

(d) As a result of shining monochromatic light with wavelength $\lambda$ on a metal surface, electrons are ejected with maximum kinetic energy $T$. If one changes the wavelength of the light to $\lambda/4$, electrons are ejected with a maximum kinetic energy $5T$. Obtain an expression for $T$ in terms of $\lambda$.

Question 2 (12 Marks)

A particle in an infinite square well has an eigenfunction:

$$\psi(x) = A \sin\left(\frac{2\pi x}{L}\right)$$

for $0 \leq x < L$ and $\psi(x) = 0$ elsewhere

(a) Calculate the expectation value of $x$.

(b) Calculate the probability of finding the particle within $\pm L/100$ of the center of the well.

(c) Calculate the probability of finding the particle within $\pm L/100$ of $x = L/4$.

(d) In a few sentences, explain the relationship between your results in (a), (b) and (c).

Question 3 (6 Marks)

(a) Does the uncertainty principle set a limit on how well you can make any single measurement of position? Briefly discuss.

(b) Suppose an electron has a kinetic energy of 3 keV, which has an uncertainty $\Delta E = 30$ eV. What will be the corresponding uncertainty in its position $\Delta x$?

(hint: obtaining the derivative $dE/dp$ might be helpful in attacking this problem.)