Question 7 18 marks

a) Assuming that \( F = -kx \) for the elastic cords, the seat (and baby) execute vertical SHM with an amplitude of 10 cm about an equilibrium position of -18 cm.

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ k = \frac{-F}{x} = \frac{12 \text{ kg} \times 9.8 \text{ m/s}^2}{0.18 \text{ m}} \]

\[ = 653 \text{ N/m} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{653 \text{ N/m}}{12+15 \text{ kg}}} \]

\[ = 0.99 \text{ Hz} \]

b) \( y = y_{\text{max}} \cos \omega t \)

\( v = \frac{dy}{dt} = -y_{\text{max}} \omega \sin \omega t \)

\( v_{\text{max}} = -y_{\text{max}} \omega = 0.18 \times 2\pi \times 0.99 \)

\[ = 1.12 \text{ m/s} \]

This occurs as the baby passes in either direction through the equilibrium position of -18 cm.

d) \( a = \frac{dv}{dt} = -y_{\text{max}} \omega^2 \cos \omega t \)

Baby is bouncer part company when \( a = g \)

\[ y_{\text{max}} = \frac{g}{\omega^2} = \frac{9.8}{(0.99 \times 2\pi)^2} = 0.25 \text{ m} \]
Question 8 18 marks

a) Aluminium  \( L_1 = \frac{n_1}{2} \cdot \lambda \),

\[ L_1 = \frac{n_1}{2} \cdot \sqrt{\frac{mg}{\rho A}} \]

But \( v_1 = \frac{x}{v} = \sqrt{\frac{mg}{\rho A}} \)

\[ L_1 = \frac{n_1}{2} \cdot \sqrt{\frac{mg}{\rho A}} \]

\[ n_1 = \frac{L_1}{\frac{1}{2} \cdot \sqrt{\frac{mg}{\rho A}}} \]

Similarly, for the steel:

\[ f = \frac{n_1}{2} \cdot \sqrt{\frac{mg}{\rho A}} \]

But \( f \) is the same in both cases.

\[ \frac{n_1}{2} \cdot \sqrt{\frac{mg}{\rho A}} = \frac{n_2}{2} \cdot \sqrt{\frac{mg}{\rho A}} \]

\[ \frac{n_2}{n_1} = \frac{1}{2}, \sqrt{\frac{\rho_2}{\rho_1}} = 2.5 \]

Smallest integer values are therefore

\[ n_2 = 5, \quad n_1 = 2 \]

\[ f = \frac{n_1}{2} \cdot \sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2 \times 0.60} \cdot \sqrt{\frac{10 \times 9.8}{2.60 \times 10^3 \times 1.10 \times 10^{-6}}} \]

\[ = 323 \text{ Hz} \]

b) \[ n_2 = \frac{n_1}{2}, \quad n_1 = 2 \]

\[ = 6 \text{ nodes} \]
Question 9 18 marks

a)

i) Car and observer have zero relative motion along line of sight.

\[ f = 12 \text{ kHz} \quad 4 \]

ii) \[ f = f_0 \left( \frac{V + V_s}{V + V_s} \right) \]

\[ V_s = \frac{750}{3.6} = 208.3 \text{ m/s} \]

\[ f = 12.0 \left( \frac{343}{343 + 89} \right) = 12.0 \times \frac{343}{432} = 9.98 \text{ kHz} \quad 4 \]

iii)

Observer

\[ 100 \text{m} \]

\[ 500 \text{m} \]

\[ 500 \text{m} \]

Car is receding from observer along a tangential line.

\[ d = \sqrt{600^2 - 500^2} \]

\[ = 332 \text{ m away} \quad 4 \]

Note: Better students might take the transit time of the sound into account. So, in part i) when the car is directly opposite the observer, the sound head was actually emitted when the car was approaching the observer and 20 m before the point of closest approach. Similarly, in ii) the car will have moved a further \[ 332 \times \frac{69.4}{343} = 67 \text{ m around the track when the observer hears the loudest note. Such students deserve extra points if not extra marks.} \]
b) \[ I = \frac{1}{2} \rho v \omega^2 s^2 \]

\[ \therefore S_{\text{max}} = \sqrt{\frac{I}{\rho v \omega^2}} \]

\[ = \sqrt{\frac{2 \times 10^{-12}}{1.2 \times 343 \times (2 \pi \times 4 \times 10^3)^2}} \]

\[ = 2.77 \times 10^{-12} \text{ m} \]

which is 100 times smaller than a molecule of air!
Question 10. 16 marks

(a) Standard form of the wave equation is

\[ E = E_0 \sin (kx - \omega t) \]  

\[ \text{Compare } E = 50 \sin \pi (4.50 \times 10^6 x - 9.00 \times 10^{14} t) \text{ V/m} \]

\[ k = \pi \times 4.50 \times 10^6 \text{ m}^{-1} \]  

\[ \text{But } \lambda = \frac{2\pi}{k} \]

\[ \therefore \lambda = 4.44 \times 10^{-7} \text{ m} \]

\[ \therefore k = 1.44 \times 10^7 \text{ m}^{-1} \]

\[ f = \frac{\omega}{2\pi} = \frac{\frac{9.00 \times 10^{14}}{2\pi}} \text{ Hz} = 4.50 \times 10^{14} \text{ Hz} \]

\[ E = 50 \text{ V/m} \]

(b) \[ \nu = \lambda f = 4.44 \times 10^{-7} \times 4.50 \times 10^{14} \]

\[ = 2.00 \times 10^8 \text{ m/s} \]

\[ \therefore \eta = \frac{c}{\nu} = 1.50 \]
Question 11  

\[ \text{Destructive interference when } 2nt = m\lambda \]

\[ \therefore \lambda = \frac{2nt}{m} = \frac{2 \times 1.30 \times 670}{m} \text{ nm} \]

\[ m = 1 \quad \lambda = 1742 \text{ nm} \]
\[ m = 2 \quad \lambda = 871 \text{ nm} \]
\[ m = 3 \quad \lambda = 581 \text{ nm} \]
\[ m = 4 \quad \lambda = 436 \text{ nm} \]
\[ m = 5 \quad \lambda = 384 \text{ nm} \]

\[ \text{Values between } 400 \text{ and } 750 \text{ nm are } 436, 581 \text{ nm.} \]
Question 12  14 marks

a) \[ d \sin \theta = m \lambda \]

: Fringe separation is \[ \Delta \sin \theta = \frac{m \lambda}{d} \]

\[ = \frac{1.00 \times 600 \times 10^{-9}}{0.5 \times 10^{-3}} \]

\[ = 1.20 \text{ mm} \]

: Minimum of diffraction pattern at \[ m = \frac{d}{\lambda a} \]

\[ \therefore m = 5 \]

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b) 2 for diagram

\[ m = 0 \]
\[ m = 1 \]
\[ m = 2 \]
\[ m = 3 \]
\[ m = 4 \]
\[ m = 5 \text{(missing)} \]
\[ m = 6 \]
\[ m = 7 \]

Phase change of \( \pi \) at see. \[ \text{2 } \frac{\pi}{2} \]

\[ \therefore \text{require path difference } \pi \text{ for maximum} \]
12(b) continued

Note to markers:
There are several geometric constructions that can be used to calculate \( \theta \). This one is from the textbook (P37.51)

\[ PR = PR' \]

\( \therefore \) The extra distance travelled by reflected ray is \( d \)

We require \( d = \frac{1}{2} \lambda = 2 \)

Now \( \lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^6} = 0.5 \text{ m} \)

\( \therefore d = 0.25 \text{ m} \)

But \( \sin \theta = \frac{d}{40 \text{ m}} \)

\( \therefore \theta = 3.58^\circ \ 