Question 1  [12 Marks]

(a) Define the average coefficient of linear expansion for a material in terms of its fractional change in length and the change in temperature. If your definition includes an equation, define all terms used in it.

(b) A circular disk has a concentric circular hole cut out of its middle, as in the diagram. Explain, giving your reasoning, what happens to both the disk and the hole as the disk is heated.

Suppose that \( r_0 = 5.0 \text{ cm} \), \( r_1 = 10 \text{ cm} \) and the coefficient of linear expansion is \( 2.0 \times 10^{-5} \degree \text{C}^{-1} \). What is the change in the area of the material that makes up the disk for a 30\(^\circ\)C increase in its temperature?

(c) Water at temperature 25\(^\circ\)C flows from a tap T into a heated container C. The container has a heating element (a resistor R) which is supplied with electrical power, \( P \), that may be varied.

The rate of flow of water \( F = 0.030 \) litres per minute. The electrical power is sufficient that the water in the container is boiling. What is the minimum power that must be supplied in the steady state so that the amount of liquid water in the container neither increases nor decreases with time? You may neglect other losses of heat, such as conduction and radiation from the container to the air.

For water, \( c = 4.200 \text{ J kg}^{-1} \text{ K}^{-1} \), \( L_{\text{vap}} = 2.30 \times 10^6 \text{ J kg}^{-1} \), \( \rho = 1,000 \text{ kg m}^{-3} \).
Question 2  [15 Marks]

(a)  Write down an equation that describes the First Law of Thermodynamics. What quantity is being conserved? Make sure that you fully describe any symbols that you use.

(b)  In the last four columns of the following table fill in the boxes with a −, + or 0 depending on whether the quantities decrease, increase, or remain (approximately) unchanged, respectively. Provide brief explanations as to your choices.

<table>
<thead>
<tr>
<th>Situation</th>
<th>System</th>
<th>Heat</th>
<th>Work Done</th>
<th>Change in Internal Energy</th>
<th>Change in Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rapidly letting air out of a</td>
<td>Air in the pump</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bicycle tire</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Covered pan of hot water sitting</td>
<td>Water in the pan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>on a bench top at room temperature</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c)  The figure shows a cylinder containing a gas which is closed by a moveable piston. The cylinder is submerged in an ice-water mixture, and the gas is initially in thermal equilibrium with the ice-water. The piston is pushed down very quickly from position 1 to position 2. It is then held at position 2 until the gas is again in thermal equilibrium with the ice-water. The piston then is slowly raised until it returns to position 1.

Sketch a Pressure-Volume diagram showing the changes that have taken place, and indicate briefly what thermodynamic processes are taking place at each stage around one complete cycle. If 200g of ice are melted during one cycle, how much work has been done on the gas?

\[ L_{\text{fusion}} = 3.33 \times 10^5 \text{ J/kg}. \]
Question 3  [15 Marks]

(a) Show that \( x = A \cos(\omega t + \phi) \) describes the displacement of a system where the restoring force is proportional to the distance from an equilibrium position, \( x \).

(b) What do the symbols \( A \), \( \omega \) and \( \phi \) correspond to?

(c) Hence show that, for a stretched spring of spring constant \( k \), with block of mass \( m \) attached at the free end, that the period of oscillation, \( T \), of the block is given by \( T = 2\pi \sqrt{\frac{m}{k}} \).

(d) For this system, give expressions for the kinetic energy and potential energy of the block at position \( x \), in terms of \( A \), \( m \), \( k \) and \( \phi \).

(e) What also is the total energy of the system at this position? Simplify your expression and comment on this value.

(f) Suppose that \( k=5.00 \text{ N/m} \) and the block has mass \( m=0.200 \text{ kg} \), and is oscillating with an amplitude of 10.0cm. If initially the velocity is 0.100 m/s in the negative \( x \)-direction, determine the equation describing the distance \( x \) from equilibrium as a function of time. What is the maximum speed and acceleration experienced by the block?

(g) If the oscillator also experiences an additional restoring force \( R = -b v \), where \( v \) is the speed and \( b \) is a positive constant, how does Newton’s Second Law describe the motion of the system? Describe qualitatively the effect on the subsequent motion of this additional force.
Question 4  [19 Marks]

(a) Two travelling waves, moving in the \(+x\) and \(-x\) directions, are described by the equations 
\[ y = C \sin(kx - \omega t) \] 
and 
\[ y = C \sin(kx + \omega t) \], respectively. What do the symbols \(k\) and \(\omega\) represent, and how are they related to the wave speed along the \(x\)-direction?

(b) Derive an expression for the wave resulting from the linear superposition of these two travelling waves. Interpret this equation quantitatively with the aid of a labelled sketch.

(c) For a thin cylindrical pipe of length \(L\), open at both ends, and sound speed \(c\), show that the frequency of the resonant modes of oscillation is given by 
\[ f = \frac{nc}{2L} \], where \(n\) is an integer, \(\geq 1\). Sketch the first three resonant modes.

(d) Suppose that a small loudspeaker with frequency adjustable from 1 to 2 kHz is placed near to, but not touching, one end of the pipe. The pipe has length 0.60m. At what frequencies will resonance occur? Take the speed of sound to be 330 m/s.

(e) If the pipe now has a small hole drilled in it, a distance \(L/3\) from one end, determine how the first three resonances will change. Sketch the first of these.

\[ \begin{align*}
&L/3 \\
\hline \\
&\text{L}
\end{align*} \]

(f) Describe how taking into account end effects will alter the resulting resonant frequencies from part (d).
Question 5  [14 Marks]

(a) Suppose that a person (the observer) is moving at speed \( v_O \) directly towards a stationary source of sound which emits waves of frequency \( f \) at wavespeed \( c \) in still air. Show that the frequency experienced by the person, \( f' \), is given by 
\[
\frac{c + v_O}{c} f.
\]

(b) Show also that a person at rest, hearing sound waves from the same source moving directly towards them at speed \( v_S \) in still air, hears that sound at a frequency \( f'' \) given by
\[
\frac{c}{c - v_S} f.
\]

(c) A person walks at 5 m/s directly towards a stationary siren emitting sound at frequency 1,500 Hz in still air. What frequency will the person hear? The speed of sound may be taken as 330 m/s.

(d) Suppose now that the siren is also moving directly away from that person at a speed of 20 m/s. What frequency will the person now experience?

(e) Suppose now that the siren is moving directly towards a wall, which is orientated perpendicular to the direction of motion. The wall reflects the sound waves back towards the observer. What frequency will the observer hear for the reflected sound waves?