Question 1  [Marks 14]

(a) Write down the equation for Coulomb’s law, defining the symbols you use. Then explain what this formula means in words.

(b) (i) Two balls of mass $m$ each have an identical charge $q$ on them. They are hung on the ends of silk threads of length $l$ as shown in the diagram, where the angle each makes with the vertical is $\theta$. By balancing the forces acting on each charge, show that their separation $x$ is given by $x = \frac{2kq^2l}{mg}$. You may assume that $\theta << 1$.

(ii) Suppose $l = 200\text{cm}$, $m = 10.0\text{g}$ and $x = 3.00\text{cm}$, then what is the charge $q$?

(iii) If the charged particles are electrons, how many excess electrons must there be on each ball?

(iv) What is the electric potential at the point midway between the two balls?

Question 2  [Marks 10]

(a) State, in words, Gauss’s law for electric fields.

(b) Suppose a non-conducting, isolated sphere of radius $R$ is uniformly filled with charge. If the total charge is $Q$ then determine the electric field at a distance $r$ from the centre of the sphere for two cases: (i) $r < R$ and (ii) $r > R$. Sketch a graph showing the variation of electric field with distance from the centre of the sphere.

(c) How would these results change if the same charge were put on a solid, conducting sphere, also of radius $R$?

(d) Show on your graph from part (b) the variation of electric field with distance in this case.
**Question 3  [Marks 10]**

(a) A positive charge moves in the opposite direction to that of a uniform electric field. Does its potential energy increase or decrease? Explain your answer.

(b) In dry conditions, with no dust in the air, what will determine the maximum potential to which the dome of a Van de Graaff generator can be raised? You may assume that the dome of the Van de Graaff generator behaves as a spherical conductor.

(c) Calculate the electric potential at a point P on the axis of the annulus shown in the figure. The annulus has a surface charge density $\sigma$. Its inner radius is $a$ and its outer radius is $b$. Point P is a distance $x$ away from the centre of the annulus.

**Question 4.  [Marks 10]**

(a) A high-voltage capacitor is charged by connecting a large potential difference across it. Why would it be dangerous to touch its terminals, even after the applied potential difference has been disconnected? What can be done to make the capacitor safe to handle after the voltage source has been removed. Write one or two clear sentences.

(b) Two identical parallel plate capacitors, each with capacitance $C$, are charged with a battery to a potential difference $V$, disconnected from the battery, and then connected in parallel. While they are still connected in parallel, the separation of the plates for one of the capacitors is halved.

(i) Find the total energy of the system of two capacitors before the plate separation is halved.

(ii) Find the potential difference across each capacitor after the plate separation on one capacitor is halved.

(iii) Find the total energy of the system after the plate separation on one capacitor is halved.

(iv) Reconcile the difference in the answers to parts (i) and (iii) with the conservation of energy.
Question 5  [Marks 14]

(a) Describe three similarities between the electric and the magnetic forces that a charged particle might experience under the influence of their fields.

(b) Describe two differences between the electric and magnetic forces that a charged particle might also experience.

(c) A conductor carrying a current $I$ in the $x$-direction passes through a region where there is a uniform magnetic field $B$ orientated perpendicular to it, as in the diagram. Describe what the electric and magnetic forces are that act upon the charge carriers, of unknown sign but of charge $q$ and moving with drift speed $v_d$.

How may the sign of the charge carriers be determined? Your answer might be a few clear paragraphs, and might include a clearly labelled sketch.

Question 6  [Marks 13]

(a) Show that a coil rotating at a constant angular frequency $\omega$ in a magnetic field $B$ can generate an emf $\varepsilon = BAN\omega \sin(\omega t)$, where $A$ is the area of the coil and $N$ is the number of turns. Provide all necessary equations, and make appropriately labelled sketches, in your answer.

(b) Draw a graph of the resulting emf as a function of time $t$. At what orientation of the coil, with respect to direction of the magnetic field, will the magnitude of the emf be (i) at a maximum and (ii) at a minimum? How does this relate to the change of magnetic flux?

(c) How could a direct current be produced using such a rotating coil? Draw a graph of the resulting emf, as in part (b).

(d) Explain why wearing a metal bracelet in a region where there is a strong, rapidly time-varying magnetic field could be dangerous.
Question 7  [Marks 11]

(a) A metal disk swings in a direction perpendicular to a region where there is a non-uniform magnetic field, as in the diagram. Air resistance and friction at the pivot are negligible. Describe what happens to the oscillations. Why does this occur, appealing to the relevant physical laws? What could be done to the disk to ensure that it continues to swing more freely?

(b) Consider a cylindrical coil with a number, \( N \), of closely spaced turns, through which a current \( I \) is flowing. Show that the inductance of the coil is given by \( L = \frac{N \Phi B}{I} \), explaining your reasoning and the symbols you use.

(c) A long solenoid is wound with a single layer of insulated copper wire, of diameter \( d = 2.50\text{mm} \). The solenoid has cross-sectional area \( A = 50\text{cm}^2 \) and \( L = 2.00\text{m} \) long, as in the diagram. Calculate the inductance of the solenoid, ignoring any edge effects. Assume that the adjacent wires touch each other and that the thickness of the insulating material around them is negligible.