Higher Physics 1A     PHYS1131 2010-T1, Solutions

Q3     27 Marks

(a) Work done on the gas \( W = -\int_{V_i}^{V_f} PdV \) where pressure and volume are \( P \) and \( V \) respectively. \( V_i \) is initial volume and \( V_f \) is final volume. NOTE THE MINUS SIGN

(b) (i) When a closed system undergoes a changes in the state the change in internal energy of the system is given by:
\[
\Delta E_{\text{int}} = Q + W
\]
where

- \( Q \) is the heat supplied to the system
- \( W \) is the work done on the system

\( \Delta E_{\text{int}} \) is a state function

(ii) For a cyclic system \( \Delta E_{\text{int}} = 0 \) as it returns to the starting point, so that \( Q + W = 0 \).

(c) (i) Work done \( W = -P\Delta V = -P(V_f - V_i) \) where \( P \) is a constant and \( V_i, V_f \) the initial and final volumes.
For an ideal gas \( PV = nRT \) where \( n \) is the # moles.
\[
\therefore V_f = \frac{nRT_f}{P} \text{ and } T_f \text{ is the final temperature.}
\]

For the liquid water \( V_i = \frac{\text{Mass}}{\text{Density}} = \frac{nM_{H_2O}}{\rho_{\text{water}}} \) where \( M_{H_2O} \) is the molar mass of water.

\[
\therefore W = -P \left[ \frac{nRT_f}{P} - \frac{nM_{H_2O}}{\rho} \right] = -nRT_f + \frac{nPM_{H_2O}}{\rho} \\
= -[2.00 \cdot 8.314 \cdot (273.16+100)] + [2.00 \cdot 1.013 \cdot 10^{15} \cdot 18/1000/(1.0 \times 10^{16}) ] \\
= -6204.9 + 3.6 \times 10^{-3} \text{ J} \\
= -6.2049 \text{ J} \text{ (i.e. the volume of the liquid water is negligible and can be neglected)} \\
= -6.20 \times 10^3 \text{ J to 3SF}
\]

(ii) Change in internal energy is given by the First Law; i.e.
\[
\Delta E_{\text{int}} = Q + W \text{ with } Q \text{ being heat supplied, } W \text{ being the work done.}
\]

\[
Q = m_{\text{water}} L_w \text{ with } L_w \text{ the latent heat of fusion} \\
= 2.00 \cdot 18 \cdot 10^{-3} \cdot 2.30 \cdot 10^6 \text{ J} \\
= 82,800 \text{ J}
\]

Thus, \( \Delta E = 82,800 - 6.201J = 76,599J = 7.66 \times 10^4 \text{ J to 3SF} \)
We know from the First Law of Thermodynamics that \( \Delta E_{\text{int}} = Q + W \).

We have \( \Delta E_{AC} = 800\text{J} \), \( W_{ABC} = -500\text{J} = W_{AB} + W_{BC} \).

From conservation of energy \( \Delta E_{ABC} = \Delta E_{AC} \) since the same initial and final states.

\[ \begin{align*}
Q_{ABC} &= \Delta E_{AC} - W_{ABC} = 800\text{J} - (-500\text{J}) = 1300\text{J} \\
\end{align*} \]

(ii) We have \( P_A = 5P_C \) \( \Delta V_{AB} = -\Delta V_{CD} \) from inspection of the \( PV \)-diagram.

Now \( W_{CD} = -P_D \Delta V_{CD} \) since \( P_D \) is fixed along the final change.

We also know that \( W_{ABC} = W_{AB} + W_{BC} = -500\text{J} \) since \( W_{BC} = 0 \) (\( V \) fixed).

\[ \begin{align*}
\therefore W_{CD} &= -P_D \Delta V_{CD} \\
&= \frac{P_A}{5} \Delta V_{AB} = \frac{-W_{AB}}{5} = -(-500/5) = +100\text{J} \\
\end{align*} \]

(iii) From the \( 1^{\text{st}} \) Law \( \Delta E_{\text{int}} = Q + W \)

\[ \Delta E_{AC} + \Delta E_{CD} + \Delta E_{DA} = 0 \text{ (cyclic)} \]

We know that with \( \Delta E_{AC} = +800\text{J} \) (given)

So that \( \Delta E_{CDA} = -800\text{J} \)

\[ \begin{align*}
W_{CDA} &= W_{CD} \text{ since } W_{DA} = 0 \text{ (Volume fixed)} \\
\text{Also } W_{CDA} &= \Delta E_{CDA} - W_{CD} = -800\text{J} - 100\text{J} = -900\text{J} \\
\end{align*} \]

This is heat taken from the system. Thus +900J is added as heat to the surroundings.
(iv) We know that $\Delta E_{DA} = +500J$

Since also $\Delta E_{DA} + \Delta E_{AC} + \Delta E_{CD} = 0$ (cyclic)

Then $500 + 800 + \Delta E_{CD} = 0$

So that $\Delta E_{CD} = -1,300J$

Also, $\Delta E_{CD} = Q_{CD} + W_{CD}$ (1st Law)

$\therefore Q_{CD} = -1,300 - 100J = -1,400J$
Block A executes SHM given by \( \ddot{x} = -\omega^2 x \) with \( \omega = 2\pi f \) and \( f = 1.50 \) Hz.

So maximum acceleration is given \( \omega^2 x_{\text{max}} \)

Friction force between A&B given by \( F = \mu_s M_B g \).

Thus \( M_B x_{\text{Max. Acceleration}} \equiv \text{Frictional Force before slip occurs} \)

\[ \therefore M_B (2\pi f)^2 x_{\text{max}} = \mu_s M_B g \]

i.e. \( x_{\text{max}} = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6 \times 9.8}{4\pi^2 (1.5)^2} = 0.0663m = 6.6\text{cm} \) to 2SF

(b) With Block C, \( \mu_s = 0.5 \)

Maximum force for friction when \( \mu_s M_C g = M_C \omega^2 x \) where \( x \) is the displacement from equilibrium, as in part (a).

\[ f_{\text{max}}^2 = \frac{\mu_s g}{4\pi^2 x_{\text{max}}} \]

i.e.

\[ \therefore f_{\text{max}} = \left( \frac{\mu_s g}{4\pi^2 x_{\text{max}}} \right)^{1/2} = \left( \frac{0.5 \times 9.8}{4\pi^2 (0.0663)} \right)^{1/2} = 1.369\text{s}^{-1} = 1.4 \text{ Hz to 2SF} \]
Q5 23 Marks
(a) The wave speed of a mechanical wave is related to the elastic properties and the inertial properties of the medium through which it is travelling, and is given by:

\[ v = \sqrt{\frac{\text{Elastic Property}}{\text{Inertial Property}}} \]

For a stretched string, of tension \( T \) and mass per unit length \( \mu \), these combine to give \( v = \sqrt{\frac{T}{\mu}} \).

(b) (i) We have a wave of form \( y = A \sin(kx - \omega t) \)

Amplitude \( A = 0.200 \text{mm} = 0.000200 \text{m} \)

Frequency \( f = 500 \text{ Hz} \) with \( \omega = 2\pi f = 1000\pi \text{ rad/s} = 3142 \text{ rad/s} \)

Wavespeed \( v = 196 \text{ m/s} = \omega/k \) so that \( k = \omega/v = 2\pi f/v = 1000\pi/196 = 16.03 \text{ m}^{-1} \)

Thus, the wave equation becomes:

\[ y = 0.00200 \sin(16.0x - 3140t) \text{ m} \] to 3SF.

(ii) We have \( v = \sqrt{\frac{T}{\mu}} \) so that

\[ T = \mu v^2 = 4.10 \times 10^{-3} \times 196^2 \text{ N} = 157.5 \text{ N} = 158 \text{ N} \] to 3SF.

(c)

Let the Tension be \( T \) at a distance \( x \) from the end, as in the diagram.

Then \( T = \mu x g = \text{Weight of string below} \ x. \)

Thus, wave speed at \( x \) is given by:
\[ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu x g}{\mu}} = \sqrt{x g} \].

\[ \therefore \frac{dx}{dt} = \sqrt{g x} \].

So \[ \int_0^x \frac{dx}{\sqrt{g x}} = \int_0^t dt \]

i.e. \( t = 2\sqrt{x/g} \) is the time to traverse a distance \( x \) along the string, since at \( t=0 \) we have \( x=0 \).

So when \( x=L \), \( \tau = 2\sqrt{L/g} \) is the time for the pulse to traverse the length of the string.

(d)

Suppose we add a mass \( M \) to the bottom of the string, as in the diagram above.

Tension at the point \( x \) is now given by

\[ T = \text{Weight of Mass} + \text{Weight of string above.} \]

i.e. \( T = Mg + \mu x g \)

\[ \frac{dx}{dt} = v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\mu} + x g} = (\frac{Mg}{\mu})^{0.5} \left[ 1 + \frac{\mu x}{M} \right]^{0.5} \]

So that \[ \int_0^x \frac{dx}{\left[ 1 + \frac{\mu x}{M} \right]^{0.5}} = \int_0^t \left( \frac{Mg}{\mu} \right)^{0.5} dt \]

\[ \left[ 2 \left[ 1 + \frac{\mu x}{M} \right]^{0.5} \left( \frac{M}{\mu} \right) \right]^{0.5} = \left( \frac{Mg}{\mu} \right)^{0.5} t \]

\[ \therefore t = 2 \left( \frac{M}{\mu g} \right)^{0.5} \left[ \left( 1 + \frac{\mu x}{M} \right)^{0.5} - 1 \right] \]
Now $\mu = m/L$ and when $x = L$ we have for the total time $\tau$:

$$\tau = 2 \left( \frac{ML}{mg} \right)^{0.5} \left\{ \left(1 + \frac{m}{M}\right)^{0.5} - 1 \right\}$$

$$= 2 \left( \frac{L}{mg} \right)^{0.5} \left\{ (M + m)^{0.5} - M^{0.5} \right\}$$
Q6  18 Marks
(a) The sinusoidal oscillator is a pure tone generator; i.e. emitting at a single frequency that is desired.

Set the two oscillators in operation, emitting their tones.

If the two oscillators emit at the same frequency you will hear a single pitch. There will be no amplitude modulation in the sound.

If the frequencies are different beats will be heard. Adjust the frequency of one oscillator; as the beat frequency decreases it approaches the frequency of the second oscillator.

(b)

![Diagram of frequency generator and string with wavelength and mass]

To get standing waves we have to have an integer number of half wavelengths if the string is fixed at both ends.

i.e. \( n \left( \frac{\lambda}{2} \right) = L \)

Suppose that \( f = v/\lambda \) with \( v = \sqrt{T/\mu} \).

Take \( n \) half wavelengths for mass \( m_n \).

Then there will be \( n+1 \) half wavelengths for a mass \( m_{n+1} \) with \( m_{n+1} < m_n \) since there are fewer nodes when the tension is more (making use of the hint) and we know that there are no standing waves between these values (given in the question).
Similarly, \( \sqrt{\frac{T_n}{\mu}} = f \frac{2L}{n+1} \) with the same frequency \( f \).

Divide these two equations:

\[
\frac{\sqrt{T_n/\mu}}{\sqrt{T_{n+1}/\mu}} = \frac{f \frac{2L}{n}}{f \frac{2L}{(n+1)}}
\]

i.e. \( \frac{T_n}{T_{n+1}} = \frac{n+1}{n} \)

But \( T_n = 25.0 g \) and \( T_{n+1} = 16.0 g \)

So \( \frac{n+1}{n} = \sqrt{\frac{25}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4} \)

Hence \( 4(n+1) = 5n \) so that \( n = 4 \). There are therefore 4 half-wavelengths.

So \( \frac{2Lf}{n} = \sqrt{T_n/\mu} \)

\[
\therefore f = \frac{4}{2 \times 1.5} \sqrt{\frac{25 \times 9.8}{0.003}} \text{ Hz on substituting.}
\]

\[
\therefore f = 381.03 \text{ Hz } = 381 \text{ Hz to 3SF}
\]

(c) Largest mass for which standing waves could be observed would have \( n = 1 \) since the greater the tension the fewer the number of modes (using the hint again).

\[
\text{i.e. } \sqrt{T_1/\mu} = \frac{f 2L}{1}
\]

\[
\therefore T_1 = \mu (2Lf)^2
\]

So that \( m_1 = T_1/g = \frac{\mu}{g} (2Lf)^2 = \frac{0.003}{9.8} \times (2 \times 1.5 \times 381.03)^2 = 399.6 \text{ kg} \)

i.e. \( m_1 = 400 \text{ kg} \) is the maximum mass possible.