TEST 2

This test is on the final sections of this session's syllabus and should be attempted by all students.

Anything written here will not be marked.
QUESTION 1

A thin non-conducting rod of length \( L \) has a charge \( -q \) uniformly distributed along its length as shown in the diagram (i).

(a) Determine the linear charge density on the rod.

(b) Determine the value of the electric field (intensity) \( E \) (magnitude and direction) at the point P as shown in the diagram, a distance \( a \) from the end of the rod.

(c) For the configuration shown in the diagram (ii), determine the value of the electric field (intensity) \( E \) (magnitude and direction) at the point P. You may use your result from part (b).
QUESTION 2  

[Marks 21]

Three points in space are labelled #1, #2 and #3 and their positions are shown on the diagram.

The points form a right-angled triangle with dimensions as shown. Initially there is no electric field in space.

(a) Calculate the work required (by an external agent) to place a charge of $+3q$ at position #1, if it is initially infinitely far from position #1.

(b) With $+3q$ at position #1, calculate the work required (by an external agent) to place a charge of $-2q$ at position #2, if it is initially infinitely far from the charge now at position #1.

(c) With $+3q$ at position #1, and $-2q$ at position #2, calculate the work required (by an external agent) to place a charge $+q$ at position #3, if it is initially infinitely far from the charges, which are now at positions #1 and #2.

(d) Calculate the electric potential energy of the charge $+q$ at position #3 with respect to infinity in the presence of the other charges now at the positions #1 and #2.

(e) Calculate the total work required (by an external agent) to assemble all three charges from their initial positions to their final positions as detailed in parts (a), (b) and (c).

(f) Determine the electric potential energy of the system of three charges when all are in their final positions as detailed in parts (a), (b) and (c).
QUESTION 3

An electron is accelerated from rest through a potential difference of 1.0 kV and directed into a region between two parallel metal plates separated by 20 mm with a potential difference of 100 V between them. The parallel plates are thin and very large compared to the separation between them and each plate has the same magnitude of surface charge density on it.

(a) Determine the velocity of the electron before it enters the parallel plates.

(b) Determine the magnitude of the electric field (intensity) $E$ between the parallel plates.

The electron now enters the region between the parallel plates moving perpendicular to the electric field between them.

(c) Draw a clear diagram showing the parallel plates, the direction of the electric field and the velocity vector of the electron on entering the plates.

A magnetic field is also applied between the parallel plates.

(d) Determine the magnetic field (induction) $B$ (magnitude and direction) which is perpendicular to both the electron path and the electric field, such that the electron will travel in a straight line. Neglect the gravitational force on the electron.

(e) Show the direction of $B$ from part (d) on your diagram in part (c).

(f) Use the Gauss Law to determine the numerical value of the surface charge density on the parallel plates. You must show all working including the Gaussian surface that you choose.
(a) State the Biot-Savart Law for magnetostatics and if you use symbols, carefully explain the meaning of each symbol.

(b) The diagram shows a circuit with curved segments that are parts of circles of radii \(a\) and \(b\). The straight segments are along the radii. A current \(i\) flows in the circuit.

Determine the magnetic field (induction) \(B\) (magnitude and direction) at the point \(P\) in terms of the given quantities in the diagram.

(c) State the Faraday Law of electromagnetic induction and if you use symbols, carefully explain the meaning of each symbol.

(d) Using a bar magnet and a loop of wire, describe in words and with the aid of diagrams, how you could physically demonstrate the electromagnetic induction effect.

(e) Apply the Faraday Law to give a detailed explanation of why the electromagnetic induction effect occurs in the demonstration you described in part (d).
This is the repeat version of TEST 1, which was held during Session.
This repeat test should be attempted by those students who missed Test 1, or
who wish to improve their mark in Test 1.

IF YOU ARE ATTEMPTING TEST 1 (Repeat):

CROSS THIS BOX

AND INDICATE YES ON THE FRONT (TITLE) PAGE.
QUESTION 5  [Marks 25]

(a) A particle executes uniform circular motion with radius $r$, as shown in the figure.

(i) From the figure, derive an expression for the position vector $\vec{r}$ in terms of the radius $r$, the angular velocity $\omega$, the time $t$, and unit vectors $\hat{i}$ and $\hat{j}$. Take the angle at time $t = 0$ to be zero.

(ii) From (i), derive the velocity $\vec{v}$ and acceleration $\vec{a}$ of the particle. Express the result for the acceleration in terms of the position vector.

(iii) Calculate the scalar product of the velocity and position vectors, $\vec{v} \cdot \vec{r}$. What does this say about the relative orientation of $\vec{v}$ and $\vec{r}$?

(b) A stone is tied to a string and swings with uniform motion in a horizontal circle. The string breaks and at a time $t_1$ later, the stone is displaced $\vec{r}_1 - \vec{r} = (5.5\hat{i} + 7.9\hat{j} - 3.1\hat{k})$ metres. (The positive z-axis is vertically up.)

(i) Find time $t_1$.

(ii) What was the speed of the stone while in circular motion?
A small block of mass $m=254\text{g}$ slides along a track as shown in the figure. The left side of the track is elevated and has a frictionless surface. From $x=a=35.0\text{cm}$, the surface is rough and flat, with a length $L=2.37\text{m}$. At the far right, at $x=L+a$, is a spring with spring constant $k=4.36\text{N/cm}$ above a frictionless surface of length $a=35.0\text{cm}$. In traversing the flat part of the track (through a distance $L$), $728\text{mJ}$ of mechanical energy is lost due to friction. The block is released from a height $h=95.0\text{cm}$ above the flat part of the track.

(a) For the first return trip to the incline, calculate:

(i) the speed of the block at $x=a$ (on first approach to the rough surface);

(ii) the maximum compression of the spring;

(iii) the maximum height on return to the incline.

(b) Find the position where the block comes to rest on the rough flat surface.
QUESTION 7  

[Marks 19]

The Sun, mass \( m = 2.0 \times 10^{30} \) kg, revolves around the centre of the Milky Way galaxy, a distance \( R = 2.2 \times 10^{20} \) m away. The Sun completes one revolution every \( 2.5 \times 10^8 \) years.

(a) Calculate the tangential speed \( v \) of the Sun about the galactic centre.

(b) From Newton's second law and Newton's law of gravitation, derive an expression for the mass of the galaxy \( M \) in terms of the Sun's tangential speed \( v \), the gravitational constant \( G \), and the distance \( R \). (Assume that the mass of the galaxy is distributed with spherical symmetry about its centre and that the Sun is at the galactic edge.)

(c) Hence, estimate the number of stars in the Milky Way. (Assume the mass of the galaxy \( M \) is comprised entirely of stars and that the mass of the Sun is typical for a star.)
QUESTION 8 [Marks 30]

(a) A 104g copper ring has a diameter of 2.54000cm at its temperature of 0 degrees Celsius. An aluminium sphere has a diameter of 2.54533cm at its temperature of 100 degrees Celsius. The sphere is placed on top of the ring, and the two are allowed to come to thermal equilibrium, no heat being lost to the surroundings. The sphere just passes through the ring at the equilibrium temperature.

(i) What is the equilibrium temperature?
(ii) What is the mass of the sphere?

Coefficients for linear expansion -
Copper: $\alpha = 17 \times 10^{-6} \text{ K}^{-1}$;  
Aluminium: $\alpha = 23 \times 10^{-6} \text{ K}^{-1}$

Specific heat capacities -
Copper: $c = 387 \text{ J/(kg.K)}$;  
Aluminium: $c = 900 \text{ J/(kg.K)}$

(b) A quantity of ideal gas occupies an initial volume $V_0$ at a pressure $P_0$ and temperature $T_0$. The gas expands to volume $V_1$. [Cautionary note: in answering questions (ii)-(iv), pay attention to signs. E.g., -5 is less than -4.]

(i) On a $PV$ diagram, graph the expansion for three cases:
(1) constant pressure (isobaric expansion),
(2) constant temperature (isothermal expansion),
(3) no heat exchange (adiabatic expansion).

(ii) In which case is the work done on the gas the greatest? The least? Present all reasoning/working.

(iii) In which case is the heat added to the gas the greatest? The least? Present all reasoning/working.

(iv) In which case is the change in internal energy $\Delta E_{\text{int}}$ the greatest? The least? Present all reasoning/working.