Q3 21 Marks

(a) Work done on the gas \( W = \int_{V_i}^{V_f} PdV \) where pressure and volume are \( P \) and \( V \) respectively. \( V_i \) is initial volume and \( V_f \) is final volume. NOTE THE MINUS SIGN

(b) (i) When a closed system undergoes a changes in the state the change in internal energy of the system is given by:

\[ \Delta E_{\text{int}} = Q + W \]

where

\( Q \) is the heat supplied to the system
\( W \) is the work done on the system and
\( \Delta E_{\text{int}} \) is a state function

(ii) For a cyclic system \( \Delta E_{\text{int}} = 0 \) as it returns to the starting point, so that \( Q + W = 0 \).

(c) (i) Work done \( W = P\Delta V = P(V_f - V_i) \) where \( P \) is a constant and \( V_i, V_f \) the initial and final volumes.

For an ideal gas \( PV = nRT \) where \( n \) is the number of moles.

\[ \therefore V_f = \frac{nRT_f}{P} \] and \( T_f \) is the final temperature.

For the liquid water \( V_l = \frac{\text{Mass}}{\text{Density}} = \frac{nM_{H_2O}}{\rho_{\text{water}}} \) where \( M_{H_2O} \) is the molar mass of water.

\[ \therefore W = -P \left[ \frac{nRT_f}{P} - \frac{nM_{H_2O}}{\rho} \right] = -nRT_f + \frac{nPM_{H_2O}}{\rho} \]

\[ = -[2.00 \cdot 8.314 \cdot (273.16 + 100)] + [2.00 \cdot 1.013 \cdot 10^5 \cdot 18/1000]/(1.0 \times 10^6) \]

\[ = -6204.9 + 3.6 \times 10^{-3} \]

\[ = -6204.9 \text{ J} \] (i.e. the volume of the liquid water is negligible and can be neglected)

\[ = -6.20 \times 10^3 \text{ J} \text{ to 3SF} \]

(ii) Change in internal energy is given by the First Law; i.e.

\[ \Delta E_{\text{int}} = Q + W \] with \( Q \) being heat supplied, \( W \) being the work done.

\[ Q = \text{mass}_{\text{water}}L_w \] with \( L_w \) the latent heat of fusion

\[ = 2.00 \cdot 18 \cdot 10^{-3} \cdot 2.30 \cdot 10^6 \text{ J} \]

\[ = 82,800 \text{ J} \]

Thus, \( \Delta E = 82,800 - 6,201J = 76,599J = 7.66 \times 10^4 \text{ J to 3SF} \)
We know from the First Law of Thermodynamics that $\Delta E_{\text{int}} = Q + W$.

We have $\Delta E_{AC} = 800\,\text{J}$, $W_{ABC} = -500\,\text{J} = W_{AB} + W_{BC}$.

From conservation of energy $\Delta E_{ABC} = \Delta E_{AC}$ since the same initial and final states.

So, $Q_{ABC} = \Delta E_{AC} - W_{ABC} = 800\,\text{J} - (-500\,\text{J}) = 1300\,\text{J}$

(ii) We have $P_A = 5P_C$, $\Delta V_{AB} = -\Delta V_{CD}$ from inspection of the $PV$-diagram.

Now $W_{CD} = -P_D \Delta V_{CD}$ since $P_D$ is fixed along the final change.

We also know that $W_{ABC} = W_{AB} + W_{BC} = -500\,\text{J}$ since $W_{BC} = 0$ ($V$ fixed).

So, $W_{CD} = -P_D \Delta V_{CD}$

$= \frac{\Delta V_{AB}}{5} \Delta V_{AB} = \frac{-W_{AB}}{5} = -\frac{-500}{5} = +100\,\text{J}$
Physics 1A PHYS1121 2010-T1, Solutions

Q4  8 Marks
(a)

Block A executes SHM given by $\ddot{x} = -\omega^2 x$ with $\omega = 2\pi f$ and $f=1.50$ Hz.

So maximum acceleration is given $\omega^2 x_{\text{max}}$

Friction force between A&B given by $F = \mu_s M_B g$.

Thus $M_B x \text{ Max. Acceleration} \equiv \text{Frictional Force before slip occurs}$

\[ \therefore M_B \left(2\pi f\right)^2 x_{\text{max}} = \mu_s M_B g \]

\[ \text{i.e. } x_{\text{max}} = \frac{\mu_s g}{4\pi^2 f^2} = \frac{0.6 \times 9.8}{4\pi^2 1.5^2} = 0.0663m = 6.6\text{cm} \text{ to 2SF} \]
Physics 1A PHYS1121 2010-T1, Solutions

Q5  17 Marks
(a) The wave speed of a mechanical wave is related to the elastic properties and the inertial properties of the medium through which it is travelling, and is given by:

\[ v = \sqrt{\frac{\text{Elastic Property}}{\text{Inertial Property}}} \]

For a stretched string, of tension \( T \) and mass per unit length \( \mu \), these combine to give \( v = \sqrt{\frac{T}{\mu}} \).

(b) (i) We have a wave of form \( y = A \sin(kx - \omega t) \)

Amplitude \( A = 0.200 \text{mm} = 0.000200 \text{m} \)

Frequency \( f = 500 \text{ Hz} \) with \( \omega = 2\pi f = 1000\pi \text{ rad/s} = 3142 \text{ rad/s} \)

Wavespeed \( v = 196 \text{ m/s} = \frac{\omega}{k} \) so that \( k = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{1000\pi}{196} = 16.03 \text{ m}^{-1} \)

Thus, the wave equation becomes:

\[ y = 0.00200\sin(16.0x - 3140t) \text{ m to 3SF.} \]

(ii) We have \( v = \sqrt{\frac{T}{\mu}} \) so that

\[ T = \mu v^2 = 4.10 \times 10^{-3} \times 196^2 \text{ N} = 157.5 \text{ N} = 158 \text{ N to 3SF}. \]

(c)

Let the Tension be \( T \) at a distance \( x \) from the end, as in the diagram.

Then \( T = \mu x g = \text{Weight of string below} \ x. \)

Thus, wave speed at \( x \) is given by:
\[ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu vg}{\mu}} = \sqrt{gx}. \]

\[ \therefore \frac{dx}{dt} = \sqrt{gx}. \]

So \[ \int_0^t \frac{dx}{\sqrt{gx}} = \int_0^t dt \]

i.e. \( t = 2\sqrt{x/g} \) is the time to traverse a distance \( x \) along the string, since at \( t=0 \) we have \( x=0 \).

So when \( x=L \), \( \tau = 2\sqrt{L/g} \) is the time for the pulse to traverse the length of the string.
Physics 1A PHYS1121 2010-T1, Solutions

Q6  14 Marks
(a) The sinusoidal oscillator is a pure tone generator; i.e. emitting at a single frequency that is desired.

Set the two oscillators in operation, emitting their tones.

If the two oscillators emit at the same frequency you will hear a single pitch. There will be no amplitude modulation in the sound.

If the frequencies are different beats will be heard. Adjust the frequency of one oscillator; as the beat frequency decreases it approaches the frequency of the second oscillator.

(b)

To get standing waves we have to have an integer number of half wavelengths if the string is fixed at both ends.

i.e. \( n \left( \frac{\lambda}{2} \right) = L \)

Suppose that \( f = v / \lambda \) with \( v = \sqrt{T/\mu} \).

Take \( n \) half wavelengths for mass \( m_n \).

Then there will be \( n+1 \) half wavelengths for a mass \( m_{n+1} \) with \( m_{n+1} < m_n \) since there are fewer nodes when the tension is more (making use of the hint) and we know that there are no standing waves between these values (given in the question).
\[ \frac{T_n}{\mu} = f\lambda_n = f \frac{2L}{n} \]

Similarly, \[ \sqrt{\frac{T_{n+1}}{\mu}} = f\lambda_{n+1} = f \frac{2L}{n+1} \] with the same frequency \( f \).

Divide these two equations:

\[ \frac{\sqrt{T_n/\mu}}{\sqrt{T_{n+1}/\mu}} = \frac{f 2L/n}{f 2L/(n+1)} \]

i.e. \[ \sqrt{\frac{T_n}{T_{n+1}}} = \frac{n+1}{n} \]

But \( T_n = 25.0 \text{g} \) and \( T_{n+1} = 16.0 \text{g} \)

So \[ \frac{n+1}{n} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} \]

Hence \( 4(n+1) = 5n \) so that \( n=4 \). There are therefore 4 half-wavelengths.

So \[ \frac{2Lf}{n} = \frac{T_n}{\mu} \]

\[ \therefore f = \frac{n}{2L} \sqrt{\frac{T_n}{\mu}} = \frac{4}{2 \times 1.5} \sqrt{\frac{25 \times 9.8}{0.003}} \text{ Hz} \] on substituting.

\[ \therefore f = 381.03 \text{ Hz} = 381 \text{ Hz to 3SF} \]