Physics 1A  PHYS1121 2006-S1 Answers

1. 10 Marks Total
(a) A neutral atom is one with no net charge. The number of electrons is the same as the number of protons.

(b) A negatively charged atom has one or more excess electrons over the number of protons.

(c) Charge 1 $+q$ at $(0,a)$, Charge 2 $+3q$ at $(2a,a)$, Charge 3 $-q$ at $(0,-a)$.

Force between charges $q_1$ and $q_2$ a distance $r$ apart is given by $F_{12} = k \frac{q_1 q_2}{r^2}$ directed towards each other if the charges have opposite signs, and away from each other if they have the same sign.

Charges 1 and 2 are $2a$ apart.

Thus $F_{12} = k \frac{3q}{(2a)^2} = 0.75k \left( \frac{q}{a} \right)^2$ directed along the positive x-axis.

Charges 1 and 3 are $\sqrt{(2a)^2 + (2a)^2} = \sqrt{8a}$ apart and the line connecting them is at $45^\circ$ to the axes.

Thus $F_{23} = k \frac{-q}{(\sqrt{8a})^2} = -0.375k \left( \frac{q}{a} \right)^2$; i.e. directed towards each other.

Thus, taking components along the x- and y-axes:

x-axis: $F_x = k \left( \frac{q}{a} \right)^2 \left[ 0.75 - 0.375 \cos(45^\circ) \right] = k \left( \frac{q}{a} \right)^2 0.485$

y-axis: $F_y = k \left( \frac{q}{a} \right)^2 \left[ -0.375 \sin(45^\circ) \right] = -k \left( \frac{q}{a} \right)^2 0.265$

Therefore, the net force on charge $3q$ is given by $F = k \left( \frac{q}{a} \right)^2 \left[ 0.485i - 0.265j \right]$. 
2. 6 Marks Total

(a) Applying Gauss’s Law, the surface must enclose a positive net total charge, since
\( \frac{q}{\varepsilon_0} = \Phi > 0. \)

(b) (i) Only the charge inside the radius \( R \) contributes to the flux.
\[ \Phi = \frac{q}{\varepsilon_0} . \]

(ii) For a sphere of radius \( 2a \) we must include the total charge from both the ring and the point charge.

\[ \text{Charge on the ring is } 2\pi a \lambda . \]

Thus, Gauss’s law gives \( \Phi = \frac{q + 2\pi a \lambda}{\varepsilon_0} . \)
3. 9 Marks Total

(a) The potential energy increases. When a charge is made to move in the direction of the field it moves to a region of lower electric potential. Then the product of the negative charge times the lower potential gives a higher potential energy.

(b) The units of linear charge density must be C/m (charge per unit length).

Thus units of $\alpha$ are \([\text{charge per unit length} / \text{length}] = \text{[charge/length]}^2 = \text{[Cm}^{-2}]\).

The electric potential is given by $V = k \frac{dq}{r}$.

Consider an element of length $dx$ at distance $x$ from the origin.

Its charge, $dq$, must be $\lambda \, dx = \alpha \, x \, dx$

and the contribution to the potential at the point $x = -D$ therefore $k \alpha \, x \, dx / (x+D)$.

Thus the total potential at the point $x = -D$ is given by (noting that $r = x+D$):

$$V = \int_0^L \frac{k \, dq}{r} = \int_0^L \frac{k \lambda \, dx}{x+D} = \int_0^L \frac{k \alpha \, x \, dx}{x+D} = k \alpha \int_0^L \frac{x \, dx}{x+D}.$$

Making use of $\int \frac{xdx}{a+1.x} = \frac{x}{1} - \frac{a}{1^2} \ln(a+1.x) = x - a \ln(a + x)$, we have

$$\int_0^L \frac{x \, dx}{x+D} = [x - D \ln(D + x)]_0^L = [L - D \ln(D + L) + D \ln(D)] = L - D \ln(1 + L/D).$$

Thus $V = k \alpha \left(L - D \ln\left[1 + L/D\right]\right)$. 

4. 10 Marks Total

(a) Nothing happens to the charge if the wires are disconnected – the capacitors remain charged.

If the wires are now connected to each other, the charges can move along the wires until the entire conductor is at a single potential, and the capacitor discharged. There is now no net charge on the capacitor.

(b) 

(i) The potential energy of a capacitor is given by \( U = 0.5CV^2 \).

Thus for the two capacitors, each charged to a potential difference \( \Delta V \), we have
\[
U = 0.5C(\Delta V)^2 + 0.5C(\Delta V)^2 = C(\Delta V)^2.
\]

(ii) The altered capacitor has capacitance \( C' = C/2 \).

The potential across each capacitor must be the same, since they are in parallel.

The total charge, \( Q \), is the same as before. Let \( \Delta V' \) be the potential difference across the capacitors after the separation.

Hence, since \( Q = CV \), we have \( Q = C\Delta V + C\Delta V \) (before) = \( C\Delta V' + C/2\Delta V' \) (after)

so that \( 2\Delta V = 3/2\Delta V' \); i.e. \( \Delta V' = 4/3\Delta V \)

(iii) The potential energy is given by \( U = 0.5CV^2 \)

so that the new potential energy is:
\[
U' = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 = \frac{16}{9}\left(\frac{1}{2} + \frac{1}{4}\right)C(\Delta V)^2 = \frac{4}{3}C(\Delta V)^2.
\]

(iv) The energy has increased by \( \left(\frac{4}{3} - 1\right)C(\Delta V)^2 = \frac{C}{3}(\Delta V)^2 \).

The extra energy comes from the work put into the system when the plates of the capacitor are pulled apart. This requires a force to be applied because the oppositely charged plates attract.
(a) Applying the right hand rule (motion up, field out of page), the proton experiences a force directed from left to right across the page, and so it veers to the right.

It will proceed to follow a circle in the clockwise direction, as it always experiences a force perpendicular to its direction of motion.

If instead the particle were a negatively-charged electron the path would veer to the left and then continue to move in a circle in the anti-clockwise direction.

At the same speed, the electron’s circle would have a much smaller radius.

(this comes from $\frac{mv^2}{r} = qvB$, hence $r = \frac{mv}{qB}$, but they don’t need to prove this)

(b) For each segment we have $I = 5.00 \text{ A}$ and $B = 0.020 \text{ T j}$.

The force on a current carrying wire is given by $\mathbf{F} = I \mathbf{l} \times \mathbf{B}$, where $\mathbf{l}$ is the vector denoting the length and direction of the wire.

Resolve $\mathbf{l}$ into components along each section of the wire.

(i) For segment $ab$  $\mathbf{l} = -0.40 \text{ m j}$. Hence $\mathbf{F} = 5.00 \times -0.40 \times 0.020 \mathbf{j} \times \mathbf{j} \mathbf{N} = 0 \mathbf{N}$.

(ii) For segment $bc$  $\mathbf{l} = +0.40 \text{ m k}$. Hence $\mathbf{F} = 5.00 \times 0.40 \times 0.020 \mathbf{k} \times \mathbf{j} \mathbf{N} = -0.040 \mathbf{i} \mathbf{N}$.

(iii) For segment $cd$  $\mathbf{l} = -0.40 \text{ m i} + 0.40 \text{ m j}$. Hence $\mathbf{F} = 5.00 \times 0.40 \times 0.020 (-\mathbf{i} \times \mathbf{j} + \mathbf{j} \times \mathbf{j}) \mathbf{N} = -0.040 \mathbf{k} \mathbf{N}$.

(iv) For segment $da$  $\mathbf{l} = +0.40 \text{ m i} - 0.40 \text{ m k}$. Hence $\mathbf{F} = 5.00 \times 0.40 \times 0.020 (\mathbf{i} \times \mathbf{j} - \mathbf{k} \times \mathbf{j}) \mathbf{N} = 0.040 (\mathbf{k} + \mathbf{i}) \mathbf{N}$.
From Ampere’s law, the magnetic field at point $a$ is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$ where $I_a$ is the net current through the area of the circle of radius $r_a$, which in this case is 1.00A out of the page.

Hence $B_a = \frac{(4\pi 10^{-7} \text{Tm}/\text{A})(1.00 \text{ A})}{2\pi(1.00 \times 10^{-3} \text{ m})} = 2.00 \times 10^{-4} = 200 \mu T$ towards the top of the page (direction from right hand rule).

Similarly, at point $b$: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$ where $I_b$ is the net current through the area of the circle of radius $r_b$, which in this case is 1.00-3.00 A = -2.00 A into the page.

Therefore $B_b = \frac{(4\pi 10^{-7} \text{Tm}/\text{A})(2.00 \text{ A})}{2\pi(3.00 \times 10^{-3} \text{ m})} = 133 \mu T$ towards the bottom of the page.
7. 8 Marks Total

(a) Faraday’s Law of Induction states that \( \varepsilon = -\frac{d\Phi}{dt} \); i.e. the emf, \( \varepsilon \), generated in a circuit is proportional to minus the rate of change of magnetic flux, \( \Phi \), through that circuit.

(b) \( \varepsilon = -\frac{d\Phi}{dt} \) with \( \Phi = BA \), and \( A \) the area of the flux linked.

Thus \( \varepsilon = -B \frac{dA}{dt} \)

where \( \frac{dA}{dt} \) is the rate of change of area = \( av \) (as only side \( a \) is cutting new flux).

So \( \varepsilon = -Bav \).

Now \( \varepsilon = IR \) for a current \( I \), so that \( IR = Bav \), taking the absolute value.

In equilibrium (i.e. constant speed, \( v \)), the weight balances the opposing force.

The force on a current carrying conductor is given \( F = Bla \),

and from Lenz’s law it must be upwards (so as to oppose the change).

Thus \( mg = Bla \) with \( I = Bav / R \) (from above).

Hence \( \frac{B^2av}{R} = mg \)

or \( B = \sqrt{\frac{mgR}{a^2v}} = \sqrt{\frac{0.5 \times 9.8 \times 2}{1^2 \times 8}} = 1.11T \).