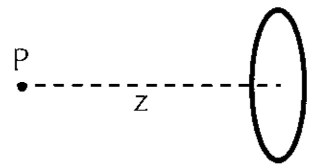
TEST 2

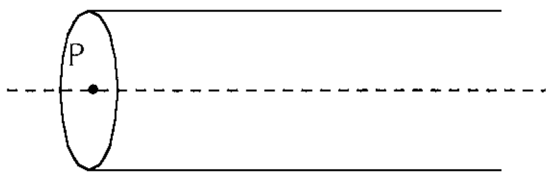
QUESTION 1 [Marks 12]

(a) A certain charge Q is divided into two parts q and Q – q, which are then separated by a certain distance. What must q be in terms of Q to maximise the electrostatic repulsion between the two charges?

(b) A thin ring of radius R carries charge q distributed uniformly around its circumference.
Determine the electric field at a point P, a distance z from the plane of the ring along its central axis.



(c) A semi-infinite cylinder of radius R is charged uniformly with a surface charge density σ. Calculate the electric field at the point P on the axis at the end of the cylinder.



QUESTION 2 [Marks 12]

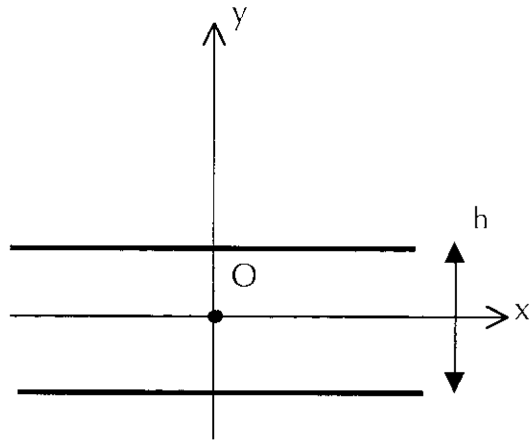
(a) Write down the formula for Gauss' law and define all symbols used.

(b) Starting from Coulomb's law and using a sphere as an imaginary surface show that Gauss' law is valid for a point-like charge.

(c) Consider a uniformly charged infinite insulating plate of thickness h.
The volume charge density is ρ.

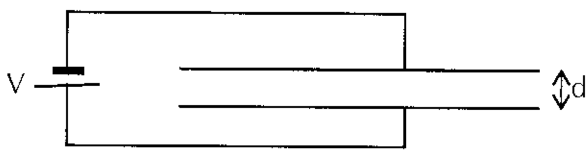
(i) Using Gauss' law find the electric field as a function of y. Clearly show the imaginary surface you use to apply Gauss' law.

(ii) Plot the electric field as a function of y.



QUESTION 3 **[Marks 11]**

(A) Two metal plates of area A are separated by a small distance d forming a parallel plate capacitor. The applied voltage between the plates is V.

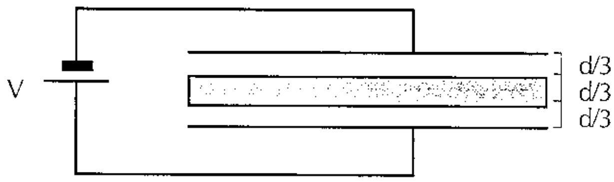


(a) Determine the electric field between the plates.

(b) Determine the electric charges on each plate.

(c) Determine the capacitance of the system.

(B) A metal slab of width d/3 is introduced between the plates as shown in the figure.



(a) Determine the electric fields at all points between the plates.

(b) Determine electric charges on both plates and on the surfaces of the metal slab.

(c) Determine the capacitance of the system.

QUESTION 4 [Marks 11]

(a) (i) Show that equivalent capacitance of two capacitors C_1 and C_2 connected in parallel is $C_{eq} = C_1 + C_2$.

(ii) Show that equivalent capacitance of two capacitors C_1 and C_2 connected in series is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

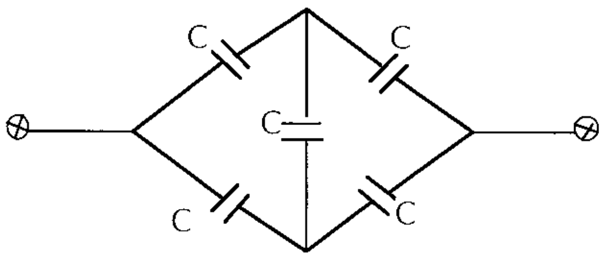
(iii) Show that equivalent inductance of two well separated inductors L_1 and L_2 connected in parallel is

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}.$$

(iv) Show that equivalent inductance of two well separated inductors L_1 and L_2 connected in series is $L_{eq} = L_1 + L_2$

(b) Determine equivalent capacitance of the circuit.

[Hint: It is not enough just to apply results of (a)i and (a)ii (capacitors in parallel and in series). You need an additional argument.]

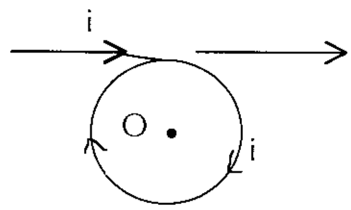


QUESTION 5 [Marks 12]

(A) Write down the formula for the Biot-Savart law and define all symbols used.

(B) Write down the formula for Ampere's law and define all symbols used.

(C) An infinite wire makes a loop of radius R (see Figure). The wire carries the electric current i. Calculate the magnetic field at the point O at the centre of the loop.
[*Hint:* This field is a combination of the field of the infinite straight wire and the field of the loop.]



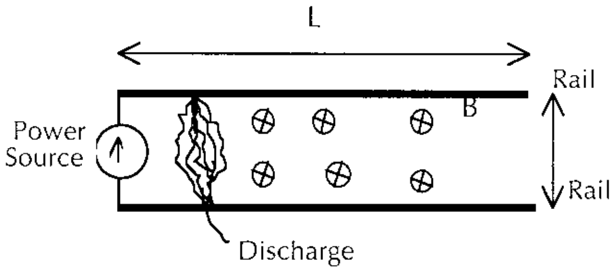
QUESTION 6 [Marks 12]

(a) (i) Using Faraday's law define what is meant by the inductance L of a coil.

(ii) Derive an expression for the total stored magnetic energy in an inductance L which is carrying a current i.

(b) A magnetic coil can withstand a maximum current i_m. At a larger current the coil is mechanically destroyed by the magnetic force. A similar coil is made from a material which is 3 times mechanically stronger. Determine the maximum current I_m that this coil can withstand. Explain your answer.

(c) A plasma accelerator consists of two metal rails in a magnetic field perpendicular to the plane (see figure).
The plasma discharge is ignited at the beginning of the accelerator and then an external power source maintains a **DC** current i through the system. The mass of the plasma cloud is m.



The parameters of the system are the following: length L = 1 m, distance between the rails ℓ = 0.1 m, current i = 10 A, field B = 1 T. The plasma discharge consists of 10^{13} Hydrogen ions, the mass of a single ion is $m_H = 1.6710^{-27}$ kg.

(i) Determine the acceleration of the plasma discharge.

(ii) Determine the final velocity of the discharge.