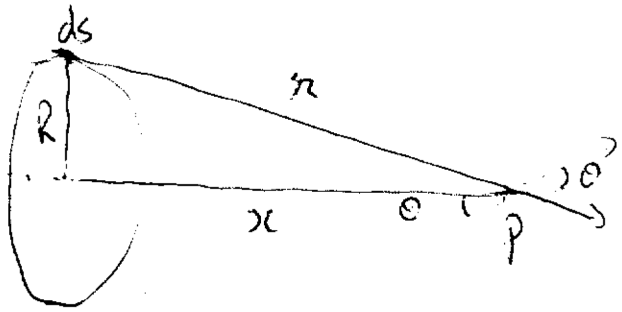
Ph 2021 Test 2 2001 Solutions

1.



(a) By symmetry the electric field at P will be in the x direction.

The field due to an element of charge $dq = \lambda\, ds$ will be in the direction shown and will have a value
$$dE = \frac{dq}{4\pi\varepsilon_0 r^2}$$ 2

The total field will then be $$\int \frac{dq \cos\theta}{4\pi\varepsilon_0 r^2}$$ 2

$$= \frac{Q}{4\pi\varepsilon_0 r^2} \cdot \frac{x}{r}$$

$$= \frac{Q\,x}{4\pi\varepsilon_0 \left(x^2 + R^2\right)^{3/2}}$$ 2

(b) If the point mass Q is at the centre of the ring, the system has a potential energy of
$$\frac{Q^2}{4\pi\varepsilon_0 R}$$
This energy is converted to kinetic energy so $\frac{1}{2} M v^2 = \frac{Q^2}{4\pi\varepsilon_0 R}$

so $$v = \sqrt{\frac{Q^2}{2\pi\varepsilon_0 M R}}$$

2. (a) Capacitor A may be considered as two capacitors in parallel, one with dielectric and one without. 1

Capacitance of A is then $C = \frac{\varepsilon \varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d}$ 2

$$= \frac{3 \times 9.85 \times 10^{-12} \times 2 \times 1.5 \times 10^{-4}}{0.5 \times 10^{-2}} + \frac{9.85 \times 10^{-12} \times 2 \times 0.5 \times 10^{-4}}{0.5 \times 10^{-2}}$$ 2

$$= \frac{9.85 \times 10^{-12} \times 10^{-4}}{0.5 \times 10^{-2}} (9+1)$$

$$= 1.97 \times 10^{-12} \sim 2\,\text{pF}.$$ 2

(b) Total capacitance of two in series

$\frac{1}{C_s} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ so $C_s = \frac{4}{3}\,\text{pF}$. 2

Charge on each capacitor $Q = C_s V = \frac{4}{3} \times 12 = 16\,\text{pC}$

P.D. across A is $V = \frac{Q}{C} = \frac{16 \times 10^{-12}}{2 \times 10^{-12}} = 8\,\text{V}$.

(c) Capacitance of A will decrease and so will total capacitance of the circuit. 1
Thus electrical energy $= \frac{1}{2} C V^2$ will decrease. 1
Energy will be lost to heat as current flows around the circuit during the change. 1

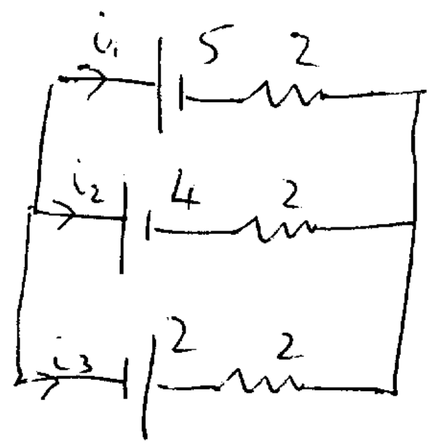
3. (a) $$R = \frac{\rho l}{A} = \frac{1.7\times10^{-8}\times 2}{\pi\left(\frac{1\times10^{-3}}{2}\right)^2}$$

$$= 0.043 \text{ ohm.}$$ 2

(b) $$\Delta R = \frac{\rho \Delta l}{A} = \frac{1.7\times10^{-8}\times 1\times10^{-3}}{\pi\left(\frac{1\times10^{-3}}{2}\right)^2}$$

$$= 2.16\times10^{-5} \text{ ohm.}$$ 2

(c) Labelling currents as shown,



$$-5 - 2i_1 + 2i_2 + 4 = 0. \quad \cdots(1)$$
$$-4 - 2i_2 + 2i_3 - 2 = 0. \quad \cdots(2)$$
$$i_1 + i_2 + i_3 = 0. \quad \cdots(3)$$

From (1) + (3): $-5 + 2i_2 + 2i_3 + 2i_2 + 4 = 0$.

or $4i_2 + 2i_3 - 1 = 0$

and (2) is $-2i_2 + 2i_3 - 6 = 0$

Subtracting $6i_2 + 5 = 0$ 2

$i_2 = -5/6$ A.

4. Force on ~~top~~ bottom part of circuit is

$$F_{BOT} = B_4 i l = \frac{4\times10^{-7}}{4\times10^{-2}} \times 5 \times 0.1$$

$$= 5\times10^{-6}\ N \quad \text{down} \qquad 2$$

Force on top part of circuit is

$$F_{TOP} = B_9 i l = \frac{4\times10^{-7}}{9\times10^{-2}} \times 5\times 0.1$$

$$= 2.2\times10^{-6}\ N \quad \text{up.} \qquad 2$$

So net force down $= 2.8 \times 10^{-6}\ N.$ 1

Force on side part of circuit

$$dF = \frac{4\times10^{-7}}{y}\ i\ dy \qquad 1$$

$$\text{or } F = 4\times10^{-7}\times i \int_{0.04}^{0.09} dy/y .$$

$$= 20\times10^{-7}\Big[\ln y\Big]_{0.04}^{0.09}$$

$$= 20\times10^{-7}\ln\left(\tfrac{9}{4}\right)$$

$$= 1.62\times10^{-6}\ N \quad \text{to left.} \qquad 2$$

5. When switch is turned to OFF, capacitor starts to be charged thru the 2 Megaohm resistor.
P.D. reached is

$$V = 12\left(1 - e^{-\frac{0.5}{1\times10^{-6}\times2\times10^{6}}}\right)$$

$$= 12\left(1 - e^{-0.5/2}\right)$$

$$= 12(1 - 0.78)$$

$$= 2.65\ V$$

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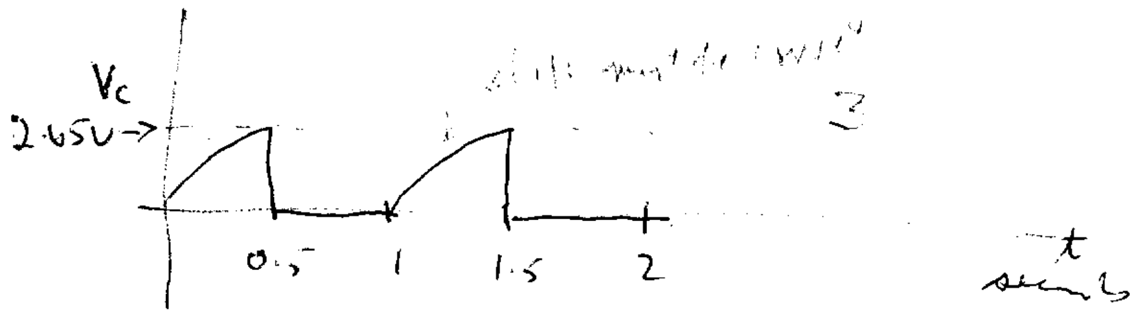

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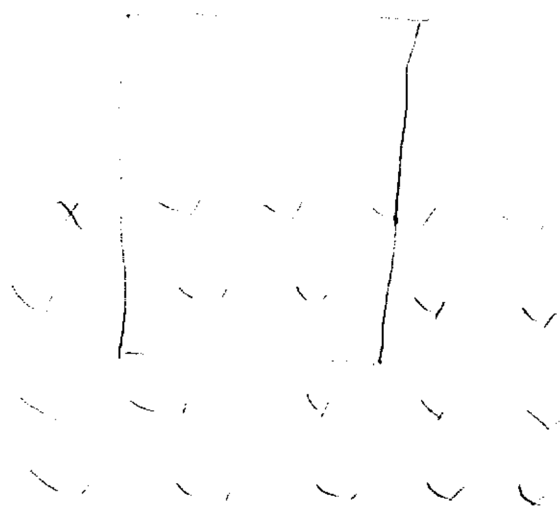
6.



(a) When the force on the loop due to the magnetic field becomes equal to the weight, the net force and acceleration goes to zero.

1

6. When wire falling vel v. $\mathcal{E} = B w v$.

$$I = \frac{B w v}{R}$$

$$F = B I w = \frac{B w \cdot B w v}{R}$$

$$= \frac{B^2 w^2 v}{R} = mg.$$

$$v = \frac{mgR}{B^2 w^2}$$

1

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