THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – JUNE 2014

PHYS2010 – MECHANICS

PHYS2120 – MECHANICS & COMPUTATIONAL PHYSICS (Mechanics Paper)

Time allowed – 2 hours

Total number of questions – 4

Answer ALL FOUR questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{c}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 e^{i\omega t}$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$
$$\theta = \pm\int \frac{(L/r^2)\,dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$

Lagrangian

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Question 1

An isotropic central force is given by:

$$\mathbf{F} = F(r)\hat{\mathbf{r}}$$

The natural coordinates for analysing central forces are plane polar coordinates (r,θ). In these coordinates, the acceleration is given by:

$$\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

where $\hat{\mathbf{r}}$ and $\hat{\theta}$ are the unit basis vectors.

(a) Write the vector equation of motion for a particle in an isotropic central force in plane polar coordinates.

(b) Split this into two scalar equations (a radial equation and an axial equation).

(c) Show that the axial equation constrains the motion of the particle. Describe the consequences of the axial equation.

The radial equation can be used to derive a differential equation describing the orbit of the particle, $r(\theta)$. To do this, one must eliminate the time derivatives from the radial equation so that the only variables are plane polar coordinates (r,θ) and derivatives of r with respect to θ.

To do this, a useful strategy is to use the following change of variable:

$$r = \frac{1}{u}$$

(d) Show that $\dot{r} = -h\dfrac{du}{d\theta}$ where *h* is the angular momentum per unit mass of the particle.

(e) Hence (or otherwise), derive an expression for $\ddot{r}$ in terms of *h, u* and its derivatives with respect to θ.

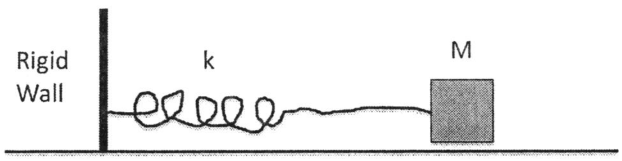
(f) Using these results (or otherwise), derive a differential equation for the orbit of a single particle of mass, *m*, in an isotropic central force field in terms of *m, h* and the variables *u* and θ.

(g) What is the form of this differential equation for the orbit when the central force is proportional to the inverse square of the radius?

Question 2

Part A

A particle of mass M is free to slide on a frictionless horizontal surface. It is then attached to a rigid wall by a spring of stiffness k.

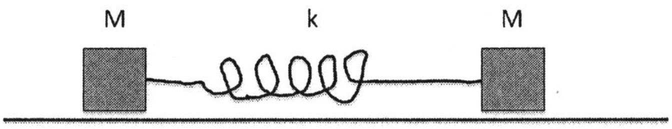


(a) Derive the equation of motion for the particle.

(b) Derive an expression for the frequency of oscillation.

Part B

Two particles, each of mass M, are free to slide on a frictionless horizontal surface. The particles are attached to each other by a spring of stiffness k.

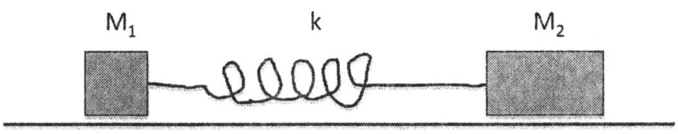


(c) Derive expressions for the equations of motion for this system.

(d) Derive an expression for the frequency of oscillation for the coupled particles

Part C

Two particles with different masses (M_1 and M_2) are free to slide on a frictionless horizontal surface. The particles are attached to each other by a spring of stiffness k.



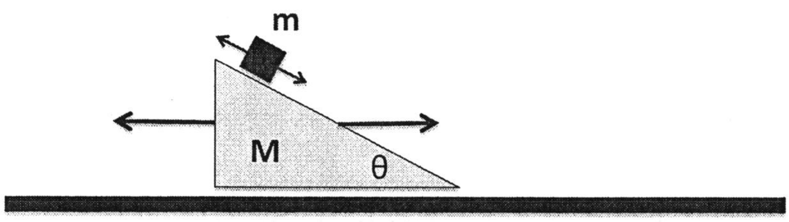
(e) Derive expressions for the equations of motion of the system.

(f) Derive an expression for the frequency of oscillation of the coupled particles.

Question 3

Part A

A solid wedge, mass M, is free to slide on a frictionless horizontal table. A second mass, m, is free to slide on the top surface of the wedge (assume no friction), which makes an angle θ to the tabletop. The system is initially held stationary before the wedge and the second mass are released under the influence of gravity.



(a) Determine the degrees of freedom of this system.

(b) Choose a set of generalised coordinates and write the Lagrangian for this system.

(c) Derive the equations of motion for the wedge and the second mass.

(d) Derive an expression for the acceleration of the wedge on the table while the second mass is still on the wedge.

Part B (Unrelated to Part A)

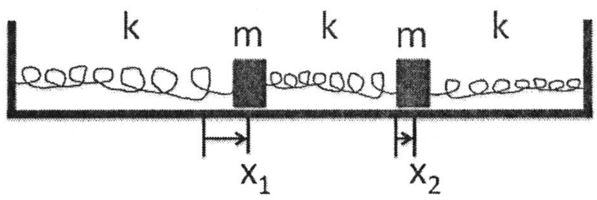
(e) Derive an expression for the moment of inertia of a solid sphere about an axis passing through its centre of symmetry.

(f) Derive an expression for the moment of inertial of a solid sphere about an axis that is tangential to the surface of the sphere.

Question 4

Part A

Two equal masses m can move on a frictionless horizontal table. They are connected by three springs each with elastic constant k. One spring connects the masses to each other while the others connect the masses to the walls.



(a) Write down the Lagrangian for this system and hence derive the equations of motion.

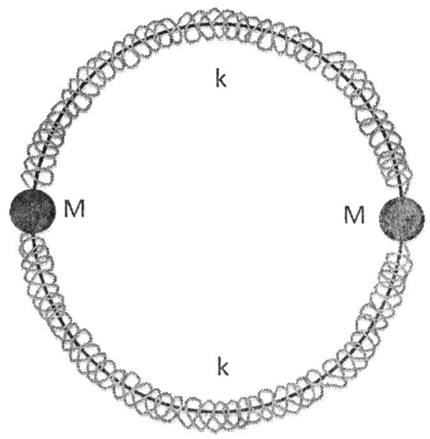
(b) Derive the secular equation for the system.

(c) Solve the secular equation and find the normal frequencies of the system.

(d) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.

Part B

Two beads of equal mass, M, are allowed to move freely on a frictionless hoop, radius R, that lies horizontally (i.e. assume gravity has no effect on the motion of the beads). Two springs are threaded onto the hoop so that they connect the beads to each other. Each spring has a stiffness k.



(e) Derive the secular equation for this system and solve it.

(f) Find the normal modes. Sketch the motions that correspond to the normal modes and indicate the corresponding normal frequencies.