Phys1131. Session 2, Test 1, 2004

Closed book examination

Time allowed - 1.5 Hours

Total number of questions -4

All questions are of equal value.

Answer ALL questions.

Question 1

All values are given in SI units.

(a). 1D problem. The initial position and velocity of a particle are $x_0 = 1$, $v_0 = 2$. The mass of the particle is $m = 3$. The particle is accelerated by the constant force $F = 9$. Calculate the acceleration $a(t)$, velocity $v(t)$ and position $x(t)$ as functions of time t.

(b). 1D problem. The initial position and velocity of a particle are $x_0 = 1$, $v_0 = 2$. The mass of the particle is $m = 3$. The particle is accelerated by the increasing time-dependent force $F = 9t$. Calculate the acceleration $a(t)$, velocity $v(t)$ and position $x(t)$ as functions of time t.

(c). 3D problem. The position of a particle of mass $m = 3$ as a function of time is given by $\mathbf{r}(t) = 3\mathbf{i} + 21\mathbf{j} + \exp(-2t)\mathbf{k}$. Calculate the velocity, acceleration and force acting on the particle as functions of time in vector form (using unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$). Then describe in words the magnitudes and directions of the velocity, acceleration and force.

Question 2

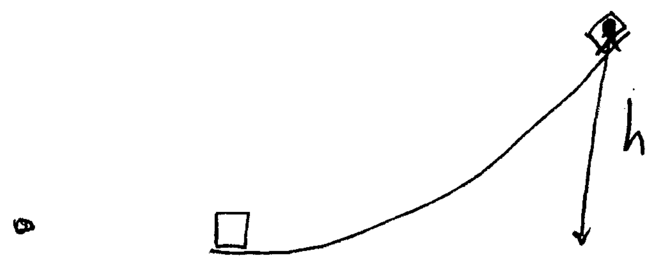
A bullet of mass $m = 10g$ and speed $v = 1000m/s$ is fired into a wooden block of mass $M = 90g$.

(a). Calculate the initial kinetic energy of the bullet.

(b). Calculate the speed of the block after the bullet is embedded in the block.

(c). Calculate the kinetic energy of the block after the bullet is embedded in the block.

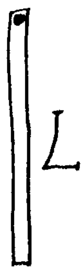
(d). After the bullet is embedded in the block it moves up a hill with a smooth surface with no friction and reaches the maximal height h. Calculate h.



(e). Now consider the same problem with the block (after the bullet is embedded) moving along a horizontal surface with a kinetic friction coefficient 0.1. Calculate the distance which the block will travel on this surface until it stops.

Question 3

A pendulum is made from a rod of mass M and length L. The axis of the pendulum is near the upper end of the rod.

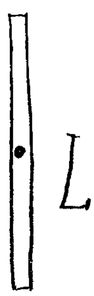


(a). Calculate the rotational inertia I of the pendulum. You may use the following formula $I = \frac{M}{L} \int x^2 dx$.

The limits of the integration depend on the position of the axis.

(b). A period of small oscillations of a pendulum which has rotational inertia I and mass M is equal to $T = 2\pi\sqrt{\frac{I}{MgL}}$. Use your result for I to calculate the period of small oscillations of the rod pendulum.

(c). Now the axis is transfered to the centre of the rod. Calculate the rotational inertia.



(d). Use your new result for I to calculate the period of small oscillations of the new rod pendulum.

Question 4

This problem describes a method to search for the dark matter. We assume that the distribution of the dark matter has a spherical symmetry. A star moves on a circular orbit around our Galaxy. The gravitational force acting on the star is proportional to the mass M_r which is located inside this circular orbit. The radius of the orbit is r, the speed of the star is v.

(a). Write an expression equating the centripetal force for the circular motion and the gravitational force between the star of mass m and the mass M_r.

(b). Use (a) to calculate the mass M_r (express M_r in terms of the gravitational constant G, the radius r and the speed v).

(c). All visible mass is concentrated near the centre of the Galaxy. Another star moves on a circular orbit at a distance R from the centre of the Galaxy with the speed v_2. Calculate the mass of the dark matter located between the radius r and radius R. Assume $R > r$.