SOLUTION to PHYS1231 FINAL EXAM S2 2010

Total for 3 Questions: 60 Marks

Question 5. EM Waves (Marks 22)

(a) A laser emits sinusoidal electromagnetic (EM) waves that travel in the negative x-direction. The EM waves of wavelength $\lambda = 10{,}600$ nm are emitted from the laser into vacuum with **E** field parallel to the z-axis; the **E** field amplitude is $1.5\text{x}10^6\ \text{Vm}^{-1}$. Write vector equations for **E** and **B** as a function of time and position. (12 marks)

Solution

The wave is travelling in the direction $-\hat{\mathbf{i}}$. The equations for E and B have the form

$$\mathbf{E}(x,t) = \hat{\mathbf{k}}E_{max}\cos(kx+\omega t)$$
$$\mathbf{B}(x,t) = \hat{\mathbf{j}}B_{max}\cos(kx+\omega t)$$

where the unit vectors $\hat{\mathbf{k}}, \hat{\mathbf{j}}$ give the orientation of **E** and **B**.

and where $E_{max} = 1.5\text{x}10^6\ \text{Vm}^{-1}$ and $B_{max} = \dfrac{E_{max}}{c} = \dfrac{1.5\text{x}10^6}{3.0\text{x}10^8} = 5.0\text{x}10^{-3}\ \text{T}$

The wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{10.6\text{x}10^{-6}\ \text{m}} = 5.93\text{x}10^5\ \text{rad m}^{-1}$

The angular frequency is $\omega = ck = (3\text{x}10^8)(5.93\text{x}10^5)\ \text{rad s}^{-1} = 1.78\text{x}10^{14}\ \text{rad s}^{-1}$

We obtain

$$\mathbf{E}(x,t) = \hat{\mathbf{k}}(1.5\text{x}10^6)\cos\left[5.93\text{x}10^5 x + 1.78\text{x}10^{14} t\right]$$

and

$$\mathbf{B}(x,t) = \hat{\mathbf{j}}(5.0\text{x}10^{-3})\cos\left[5.93\text{x}10^5 x + 1.78\text{x}10^{14} t\right]$$

(b) In a CD ROM drive light from a semiconductor diode laser having wavelength $\lambda = 780nm$ travels a distance 125 nm in a polycarbonate layer, Polycarbonate is a transparent medium of refractive index 1.58. Calculate,

(i) the optical path length (2 marks)
(ii) the wavelength of the light in the transparent medium (3 marks)
(iii) the phase difference after travelling the distance 125nm with respect to light travelling the same distance in free space. (5 marks)

Solution

(i) the optical path length (o.p.l) is

o.p.l = (physical distance travelled in medium) x (refractive index)

$$= \left(125\text{x}10^{-9}\ \text{m}\right)\left(1.58\right) = 1.97\text{x}10^{-7}$$

(ii) The wavelength in the polycarbonate medium is λ_m given by

$$\lambda_m = \frac{\lambda_0}{n} = \frac{780\ \text{nm}}{1.58} = 493.7\text{nm}$$

where λ_0 is the free space (vacuum) wavelength.

(iii) The physical distance travelled is $l = 125\text{x}10^{-9}\text{m}$

The refractive indices are: $n_{medium} = 1.58,\ n_{vacuum} = 1.0$

$\therefore$ path difference = $(n_{medium} l - n_{vacuum} l)$

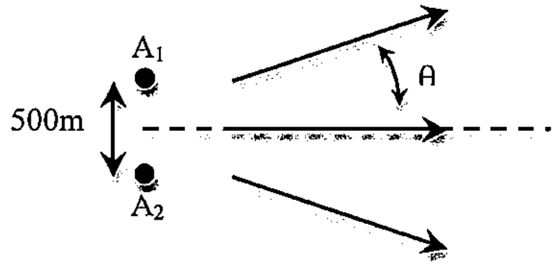
$$= (1.58)\left(125\text{x}10^{-9}\right) - (1.0)\left(125\text{x}10^{-9}\right) = 7.25\text{x}10^{-8}\ \text{m}$$

Phase difference = 2π(path difference/wavelength)

$$= 2\pi\left(\frac{7.25\text{x}10^{-8}}{780\text{x}10^{-9}}\right) = 0.58\ \text{radians}$$

Question6. (Marks 18)

(a) A pair of antennas, A_1 and A_2, spaced 500 m apart broadcast a radio signal at frequency 1200 kHz. The signals broadcast from the antennas are of equal power and in phase. Calculate the angular directions θ in which the resultant intensity in the radiation pattern is greatest. (8 marks)



Solution

Maxima in the resultant intensity are given by

$\sin\theta = \dfrac{m\lambda}{d}$ (m = 0, ± 1, ± 2...) and d is the spacing between the aerials.

The wavelength is $\lambda = \dfrac{c}{f} = \dfrac{3.0\text{x}10^8}{1200\text{x}10^3} = 250\text{ m}$

Then,

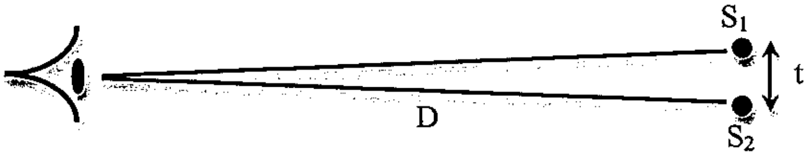
$m = 0 \rightarrow \sin\theta = 0 \rightarrow \theta = 0$

$m = \pm 1 \rightarrow \sin\theta = \pm\dfrac{250}{500} \rightarrow \pm 30^\circ$

$m = \pm 2 \rightarrow \sin\theta = \pm\dfrac{500}{500} \rightarrow \pm 90^\circ$

or in terms of the range 0 - 2π radians stated in the question.
0, π/3, π/2, 5π/6, 7π/6, 3π/2, 11π/6.

(b) A pair of closely spaced light sources S_1 and S_2 separated by distance t, are viewed by the eye. The sources emit light with wavelength λ=550nm. The eye is D metres from the light sources, as shown schematically below. Assuming diffraction limited resolution, determine the minimum spacing t for which S_1 and S_2 may be clearly resolved as two separate sources if D = 1200 m. The diameter of the pupil of the eye may be taken to be 3mm. (10 marks)



Solution

$$D = 1200\ \text{m} = \frac{tP}{1.22\lambda} \approx \frac{t\left(3.0\text{x}10^{-3}\,\text{m}\right)}{1.22\left(550\text{x}10^{-9}\right)}$$

$$t = \frac{(1200)(1.22)\left(550\text{x}10^{-9}\right)}{3.0\text{x}10^{-3}\,\text{m}} = 0.27\ \text{m}$$

Question 7 (20 Marks)

(a) A sodium atom emits a photon of wavelength 589.0 nm and energy 2.105 eV in a transition from an excited state to the ground state. The atom remains in the excited state for an average 'lifetime' $\tau = 0.16$ ns before the transition to the ground state. Calculate,

(i) the uncertainty in the energy of the excited state (4 marks),

(ii) the width (i.e. the spread in wavelength) of the line in the observed spectrum associated with this transition. (5 marks)

Solution

$$\text{(i) } \Delta E = \frac{\hbar}{\Delta t} = \frac{1.05\text{x}10^{-34}}{0.16\text{x}10^{-9}} = 6.6\text{x}10^{-25}\,\text{J} = 4.1\text{x}10^{-6}\,\text{eV}$$

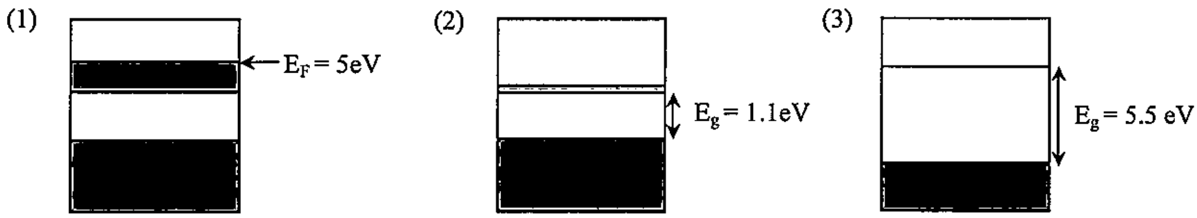
note: also correct to use $\Delta E \Delta t \geq \frac{\hbar}{2}$.

Solution

$$\text{(ii) uncertainty in photon energy } = \frac{4.1\text{x}10^{-8}\text{ eV}}{2.105\text{ eV}} = 1.95\text{x}10^{-8}$$

$$\therefore\ \Delta\lambda = \left(1.95\text{x}10^{-8}\right)\left(589.0\text{ nm}\right) = 1.1\text{x}10^{-5}\text{ nm}$$

(b) Three materials have the energy band structures shown schematically in the diagram below representing, (1) a metal, (2) an n-type doped semiconductor and (3) an insulator. The shaded areas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity and the thermal velocity of the electrons at 300K. (4 marks)

(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2). (2 marks)

(iii) By comparing the energy gap values for materials (2) and (3) state, with your reasoning, whether material (3) is expected to be transparent or opaque to visible light at room temperature. (The visible region of the EM spectrum spans the wavelength range λ=400nm to λ=700nm approx.) (5 marks)

Solution

(i) Fermi velocity

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2(5.0 \times 1.602 \times 10^{-19} J)}{9.1x10^{-31} kg}} \approx 1.3x10^6 \ ms^{-1}$$

Thermal velocity:

$$v_{th} = \sqrt{\frac{2k_B T}{m_e}}$$

$$= \sqrt{\frac{2(1.38x10^{-23} \ JK^{-1})(300K)}{9.1x10^{-31}}}$$

$$\approx 1x10^5 ms^{-1}$$

(ii)

$$\lambda_g = \frac{hc}{E_g} = \frac{(6.63x10^{-34})(3x10^8)}{(1.1)(1.6x10^{-19})} = 1.1x10^{-8} m$$

(iii) **Solution**

Material (1) is a metal and will retain metallic conductivity at low temperature and as T tends to 0K.

Material (2) is a doped semiconductor and will become an insulator as T approached 0K as the free electrons in the conduction band 'freeze out'.