SOLUTION to PHYS1221 FINAL EXAM S2 2010

Total for 3 Questions: 45 Marks

Question 4. EM Waves (Marks 14)

(a) A laser emits sinusoidal electromagnetic (EM) waves that travel in the negative x-direction. The EM waves of wavelength $\lambda = 10{,}600$ nm are emitted from the laser into vacuum with **E** field parallel to the z-axis; the **E** field amplitude is $1.5\text{x}10^6\ \text{Vm}^{-1}$. Write vector equations for **E** and **B** as a function of time and position. (8 marks)

Solution

The wave is travelling in the direction $-\hat{\mathbf{i}}$. The equations for E and B have the form

$$\mathbf{E}(x,t) = \hat{\mathbf{k}}E_{max}\cos(kx + \omega t)$$
$$\mathbf{B}(x,t) = \hat{\mathbf{j}}B_{max}\cos(kx + \omega t)$$

where the unit vectors $\hat{\mathbf{k}}, \hat{\mathbf{j}}$ give the orientation of **E** and **B**.

and where $E_{max} = 1.5\text{x}10^6\ \text{Vm}^{-1}$ and $B_{max} = \frac{E_{max}}{c} = \frac{1.5\text{x}10^6}{3.0\text{x}10^8} = 5.0\text{x}10^{-3}\ \text{T}$

The wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{10.6\text{x}10^{-6}\ \text{m}} = 5.93\text{x}10^5\ \text{rad.m}^{-1}$

The angular frequency is $\omega = ck = \left(3\text{x}10^8\right)\left(5.93\text{x}10^5\right)\text{rad}\,s^{-1} = 1.78\text{x}10^{14}\ \text{rad}\,s^{-1}$

We obtain

$$\mathbf{E}(x,t) = \hat{\mathbf{k}}\left(1.5\text{x}10^6\right)\cos\left[5.93\text{x}10^5 x + 1.78\text{x}10^{14} t\right]$$

and

$$\mathbf{B}(x,t) = \hat{\mathbf{j}}\left(5.0\text{x}10^{-3}\right)\cos\left[5.93\text{x}10^5 x + 1.78\text{x}10^{14} t\right]$$

(b) In a CD ROM drive light from a semiconductor diode laser having wavelength $\lambda = 780nm$ travels a distance 125 nm in a polycarbonate layer, Polycarbonate is a transparent medium of refractive index 1.58. Calculate,

(i) the optical path length (3 marks)

(ii) the wavelength of the light in the transparent medium (3 marks)

Solution

(i) the optical path length (o.p.l) is

o.p.l = (physical distance travelled in medium) x (refractive index)

$= (125\text{x}10^{-9}\ \text{m})(1.58) = 1.97\text{x}10^{-7}$

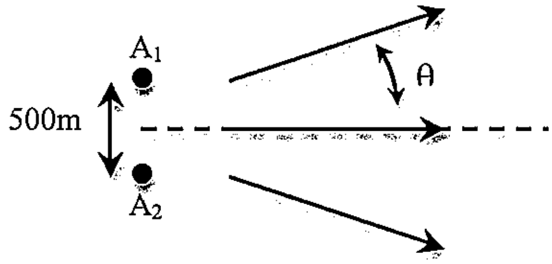
(ii) The wavelength in the polycarbonate medium is λ_m given by

$$\lambda_m = \frac{\lambda_0}{n} = \frac{780\ \text{nm}}{1.58} = 493.7\text{nm}$$

where λ_0 is the free space (vacuum) wavelength.

Question 5. (Marks 12)

A pair of antennas, A_1 and A_2, spaced 500 m apart broadcast a radio signal at frequency 1200 kHz. The signals broadcast from the antennas are of equal power and in phase. Calculate the angular directions θ in which the resultant intensity in the radiation pattern is greatest. (12 marks)



Solution

Maxima in the resultant intensity are given by

$$\sin\theta = \frac{m\lambda}{d} \quad (m = 0, \pm 1, \pm 2...)$$ and d is the spacing between the aerials.

The wavelength is $\lambda = \frac{c}{f} = \frac{3.0\text{x}10^8}{1200\text{x}10^3} = 250\text{ m}$

Then,

$$m = 0 \rightarrow \sin\theta = 0 \rightarrow \theta = 0$$

$$m = \pm 1 \rightarrow \sin\theta = \pm\frac{250}{500} \rightarrow \pm 30^\circ$$

$$m = \pm 2 \rightarrow \sin\theta = \pm\frac{500}{500} \rightarrow \pm 90^\circ$$

using the range in the question $0 - 2\pi$ radians:

$0, \pi/3, \pi/2, 5\pi/6, 7\pi/6, 3\pi/2, 11\pi/6.$

Question 6 (19 Marks)

(a) For an electron confined to a region of width 0.05 nm estimate the minimum uncertainty in the x-component of the electron's momentum. (5 marks)

Solution

$$(\Delta p_x)_{min}(\Delta x) = \frac{h}{2\pi}$$

note: can also use $\Delta p \Delta x = \frac{\hbar}{2}$.

$$(\Delta p_x)_{min} = \frac{1.05\text{x}10^{-34}}{0.05\text{x}10^{-9}} = 2.1\text{x}10^{-24}\ \text{kgms}^{-1}$$

(b) Calculate the de Broglie wavelength of a 25kV electron. State whether relativistic corrections are significant? (Support your statement with a numerical estimate.) (3 marks)

Solution

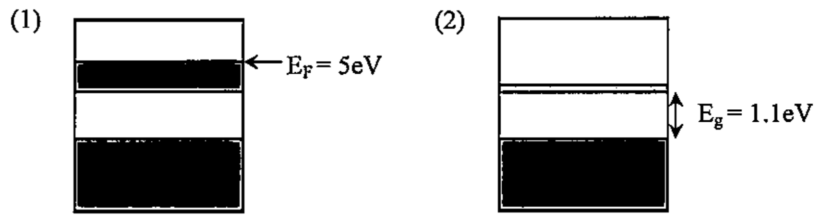
$$eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.6\text{x}10^{-19})25\text{x}10^3}{9.11\text{x}10^{-31}}} = 9.37\text{x}10^7\ \text{ms}^{-1}$$

- -this is a non-relativistic case.

$$\lambda = \frac{h}{p} = \frac{6.63\text{x}10^{-34}}{(9.11\text{x}10^{-31})(9.37\text{x}10^7)} = 7.77\text{x}10^{-12}\ \text{m} \quad (= 7.77\text{pm})$$

(c) Two materials have the energy band structures shown schematically in the diagram below representing (1) a metal and (2) an n-type doped semiconductor. The shaded areas indicate occupied (by electrons) energy ranges.



(i) For the metal shown in (1), find the Fermi velocity. (3 marks)

Solution

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2(5.0 \times 1.602 \times 10^{-19} J)}{9.1x10^{-31} kg}} \approx 1.3 \times 10^6\ ms^{-1}$$

(ii) Find the wavelength of EM radiation that will cause a sharp increase in the electrical conductivity of material (2). (4 marks)

Solution

$$\lambda_g = \frac{hc}{E_g} = \frac{(6.63x10^{-34})(3x10^8)}{(1.1)(1.6x10^{-19})} = 1.1x10^{-8}m$$

(iii) Comment on the expected electrical conductivity of materials (1) and (2) at very low temperatures, as the temperature tends towards 0K. (4)

Solution

Material (1) is a metal and will retain metallic conductivity at low temperature and as T tends to 0K.

Material (2) is a doped semiconductor and will become an insulator as T approached 0K as the free electrons in the conduction band 'freeze out'.