PHYS1221/1231 Final Exam November 2011

(Part 2)

Question 4 (15/20 marks)

Solar radiation falling normally on the Earth's surface has a power of 1.37 kW/m^2.

(a) Calculate the *maximum* magnitude of the electric and magnetic fields in this radiation.

(b) What is the pressure on a surface that completely absorbs this radiation?

(c) (1231 only) How many photons strike each square metre per second? (Assume the radiation is all at a single wavelength of 550 nm.)

Question 5 (15/20 marks)

Two transmitters, producing microwaves of wavelength 5 cm, with identical polarization, are placed a distance 20 m apart. A receiver can move freely along the line joining them. You may assume the receiver is close to the mid-point, so that the waves are plane-parallel and the amplitudes of the two waves can be taken as equal.

(a) Write an equation for the electric field (maximum value $2E_0$) as a function of position (x) along the line joining the transmitters, and the time (t). How far apart are the maxima of intensity observed in the receiver?

(b) How far do these maxima move if a slab of wax (n=1.5 at this wavelength) of thickness 2 cm is placed in front of one of the transmitters?

(c) (1231 only) Suppose, in the original set-up (ie without the wax), one transmitter is polarized horizontally, the other vertically. Describe in general terms how the polarization of the received signal varies as the receiver is moved along the line.

Question 6 (15/20 marks)

(a) Many streetlights use sodium atoms to give (yellow) light of wavelength $\lambda = 588\,\text{nm}$, in a very sharp spectral line. However, newer versions use sodium at high pressure, where the line is broadened to $\Delta\lambda = 34\,\text{nm}$. Calculate the average time between collisions of the sodium atoms.

(b) Calculate the wavelength of (i) an electron, and (ii) a photon, each of energy 10 eV.

(c) (1231 only) Calculate the speed of the electron in part (b).

OUTLINE ANSWERS

Q.4

(a) $I = 1.37\times10^3 = S_{av} = \dfrac{E_{max}^2}{2\mu_0 c} = \dfrac{cB_{max}^2}{2\mu_0}$

$\therefore\ E_{max}^2 = 2\mu_0 cI = 2\times4\pi\times10^{-7}\times3\times10^8\times1.37\times10^3 = 1.033\times10^6 \quad \therefore\ E_{max} = 1.016\times10^3 = 1.02\,\mathrm{kV/m}$

$B_{max} = E_{max}/c = 0.339\times10^{-5} = 3.39\,\mu\mathrm{T}$

(b) Pressure $= I/c = 1.37\times10^3/3\times10^8 = 4.57\times10^{-6}\,\mathrm{Pa}$

(c) For one photon, $E = h\nu = \dfrac{hc}{\lambda} = \dfrac{6.63\times10^{-34}\times3\times10^8}{550\times10^{-9}} = 3.62\times10^{-19}\mathrm{J}$

$\therefore$ number of photons, $n = \dfrac{1.37\times10^3}{3.62\times10^{-19}} = 3.78\times10^{21}$ per m^2 per second

Q.5

(a) $E_1 = E_0\sin(kx-\omega t) \quad E_2 = E_0\sin(kx+\omega t) \qquad \therefore\ E = E_1+E_2 = 2E_0\,\sin kx\,\sin\omega t$

Or: $E = 2E_0\,\sin\left(\dfrac{2\pi x}{\lambda}\right)\cos(2\pi f t)$

Since: $\lambda = 0.05\,\mathrm{m}$, $k = \dfrac{2\pi}{0.05} = 125.7\,\mathrm{rad\,m^{-1}}$,

and $\omega = ck = 3\times10^8\times125.7 = 3.77\times10^{10}\,\mathrm{rad\,s^{-1}}$, $\quad f = \dfrac{c}{\lambda} = \dfrac{3\times10^8}{0.05} = 6\times10^9\,\mathrm{Hz} = 6\,\mathrm{GHz}$

$\therefore\ E = 2E_0\,\sin(125.7x)\,\sin(3.77\times10^{10}t)$

Maxima of intensity occur at $2\pi/\lambda = n\pi$, ie $x = \lambda/2$, ie every 2.5 cm.
NB This is just like standing waves: anti-nodes every half wavelength.

(b) OPL of the wax $= nL = 0.02\times1.5 = 0.03$ m. (Previously 0.02 m.) So extra OPL is 0.01 m.
Effectively, one transmitter is now 0.01 m (=1 cm) further away. $\therefore$ shift of interference pattern by half this = 0.5 cm.

(c) Polarization is linear at ±45° when signals are in/out of phase. In fact, as the receiver is moved, the polarization goes through the cycle of linear (+45°) – elliptical – circular – elliptical – linear (−45°) – elliptical – circular (but opposite sense) – etc. NB The *intensity* remains constant.

Q.6

(a) $\lambda_1 = 588+17 = 605\,\mathrm{nm}$, $\quad \lambda_2 = 588-17 = 571\,\mathrm{nm} \quad \therefore\ E_1 = \dfrac{hc}{\lambda} = \dfrac{1240}{605} = 2.05\,e\mathrm{V}$, $\quad E_2 = \dfrac{1240}{571} = 2.17\,e\mathrm{V}$

$\therefore\ \Delta E = 0.12\,e\mathrm{V} = 1.95\times10^{-20}\mathrm{J} \quad \therefore\ \Delta t \approx \dfrac{\hbar/2}{\Delta E} = \dfrac{6.63\times10^{-34}}{4\pi\times1.95\times10^{-20}} = 2.70\times10^{-15}\,\mathrm{s}$

(b) NB $E = 10\ e\mathrm{V} = 1.6\times10^{-18}$ J

For the electron, $\lambda = \dfrac{h}{p}$, where $p = \sqrt{2mE}$

So $p = \sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-18}} = \sqrt{2.91\times10^{-48}} = 1.71\times10^{-24}\,\mathrm{kg\,m\,s^{-1}}$

$\therefore\ \lambda_e = \dfrac{6.63\times10^{-34}}{1.71\times10^{-24}} = 3.88\times10^{-10}\,\mathrm{m}$

For the photon $\lambda_\gamma = \dfrac{hc}{E} = \dfrac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-18}} = 1.24\times10^{-7}\,\mathrm{m}$

(c) $p = mv \quad \therefore\ v = \dfrac{p}{m} = \dfrac{1.71\times10^{-24}}{9.11\times10^{-31}} = 1.88\times10^6\,\mathrm{ms^{-1}}$