Question 1

a) Lower, since $f \propto \frac{1}{\sqrt{m}}$

b) $f_0 = \frac{1}{2\pi}\sqrt{k/m}$ — (1)

$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m+m_1}}$ — (2)

$\therefore \; f_0/f_1 = \sqrt{k/m} \Big/ \sqrt{\frac{k}{m+m_1}} = \sqrt{\frac{m+m_1}{m}}$

$\therefore \; m\left(f_0/f_1\right)^2 = m + m_1$

$\therefore \; m = \frac{m_1}{(f_0/f_1)^2 - 1}$

OR

$m = \frac{m_1 f_1^2}{f_0^2 - f_1^2}$

c) From (1), $k = 4\pi^2 f_0^2 m$

$\therefore \; k = \frac{4\pi^2 f_0^2 f_1^2 m_1}{f_0^2 - f_1^2}$

d) For closed pipe, $L = \lambda/4$

But $\lambda = v/f$

$\therefore \; L = v/4f = \frac{343 \text{ m/s}}{4 \times 20 \text{ Hz}} = 4.29 \text{ m}$

e) For open pipe, $L = \lambda/2 = 8.56 \text{ m}$

Question 2

a) 440 Hz. The frequency is unchanged because the observer and source are not moving relative to each other

b) $f = f_0 \left(\frac{v + v_0}{v + v_s} \right)$

Consider the frame of reference of the wind. The source is receeding at 20 ms^{-1} and the observer is approaching at $(20 - 15) = 5\ ms^{-1}$

$$\therefore f = \left(\frac{343 + 5}{343 + 20} \right) \cdot 440$$

$$= 422\ Hz.$$

c) Let I_1 be intensity of trumpet at 5 m and I_2 be ambient noise level.

$$\beta = 10 \log_{10} \left(I/I_0 \right)$$

$\therefore$ ratio of (trumpet at 5 m) to (ambient noise) is given by

$$(86 - 40)\ dB = \log_{10} \left(I_1/I_2 \right)$$

$$\therefore I_1/I_2 = 10^{4.6}, \quad I_1 = 39,800\ I_2$$

But $I = P/4\pi r^2$

$$\therefore I_1/I_2 = \left(\frac{r_2}{r_1} \right)^2$$

$$\therefore r_2 = 998\ m \ (\text{say } 1\ km)$$

$v = 15 \, ms^{-1}$

$\therefore$ time to travel $(998 - 5)\,m$ is 66 sec.

Assumptions:

- Sound radiated uniformly in all directions
- Trumpet inaudible when sound level equals ambient noise
- No attenuation of sound through air
- Listener accelerates instantly from rest.
- etc.

Question 3.

$$y = 0.5 \sin(\pi x/3) \cos(40\pi t) \quad \text{—} (1)$$

a) At $x = 4.5$ cm,

$$y_{max} = 0.5 \sin(3\pi/2) = 0.5 \text{ cm}$$

b) Nodes when $\pi x/3 = n\pi$, ie $x = 3n$

ie, $x = 0, 3, 6, 9$ cm

$\therefore \Delta x = 3$ cm

c) Maximum acceleration at antinodes, ie where $x = 3(n + 1/2)$ $\quad \pi x/3 = (n + 1/2)\pi$

$\therefore x = 1.5,\ 4.5,\ 7.5$ cm

d) $$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

compare to (1)

$\therefore$ amplitude of each wave $= 0.5/2 = 0.25$

sin term: $\frac{A+B}{2} = \pi x/3 \quad$ — (2)

cos term $\frac{A-B}{2} = 40\pi t \quad$ — (3)

Add (2) + (3) $\therefore A = \pi x/3$

Subtract (3) from (2) $\therefore B = 40\pi t$

$\therefore$ two waves are $y_1 = 0.25 \sin(\pi x/3 + 40\pi t)$

and $y_2 = 0.25 \sin(\pi x/3 - 40\pi t)$

e) $v = \omega/k$

$\omega = 40\pi\ ,\quad k = \pi/3$

$\therefore\ v = 120\ cm/s$

But $v = \sqrt{T/\mu}$

$\therefore\ T = (1.2)^2 .\ 0.024/0.09$

$= 0.38\ N$

Question 4.

a) Phase change at first surface is π, since $\sqrt{n_1} > 1$
Phase change at second surface is π, since $n_1 > \sqrt{n_1}$

$\therefore \; 2nt = \lambda/2$ where $n = \sqrt{n_1}$

$$\therefore t = \frac{550}{4\sqrt{1.55}} = 110 \text{ nm}$$

b) At shorter wavelengths (and longer wavelengths) the coating will not be the optimum thickness, and will reflect more incident blue (and red) light.

c) Now phase change at first surface is 0, since $1.7 > \sqrt{1.55}$

$\therefore$ Coating is no longer optimum (in fact it is the worst possible thickness!)

Optimum thickness is $2nt = \lambda$

$$\therefore t = \frac{550}{2\sqrt{1.55}} = 220 \text{ nm}$$

Question 5.

a) $\theta = 1.22\, \lambda / D$

$$\lambda = v/f = \frac{3.00 \times 10^8}{2.2 \times 10^9} = 0.136 \text{ m}$$

$\therefore \quad \theta = 2.56 \times 10^{-3}$ radians

b) $d = \theta r = 2.60 \times 10^{-3} \times 3.80 \times 10^5$ km

$= 988$ km

(Also acceptable is $\theta \approx \lambda / D$, giving $\theta = 2.13 \times 10^{-3}$, $d = 808$ km)

Question 6.

a) The sky is most highly polarised when the angle formed by sun - (patch of sky) - observer is 90°.

i.e. just after sunrise and just before sunset.

b) For maximum polarisation, the sun must be incident on the lake at Brewster's angle, given by

$$\tan \theta_p = n$$

For $n = 1.33$, $\theta_p = 53°$

At the equator, the sun moves at 15°/hour (360° in 24 hours). Therefore it is 53° from zenith at noon ± 3.5 hours, or 8:30 am and 3:30 pm.