THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – MAY 2007

PHYS4103 – PHYS IV (HONOURS)
UNIT D ELECTROMAGENTISM
AND THE STANDARD MODEL

Time Allowed – 3 hours

Total Number of Questions – 4

All questions are of equal value

All questions should be attempted

Candidates may bring their own calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

This paper may be retained by the candidate

Electromagnetism and Standard Model
Exam 2007

All 4 questions have same value 25%.

Part 1

I. Lorentz invariance (25%)

Suppose there are two reference frames K and K', and K' moves in relation to K with velocity v in the x-direction.

1. Verify that the Lorentz transformations[1]

$$t'=\frac{t-vx/c^2}{\sqrt{1-v^2/c^2}},$$
$$x'=\frac{x-vt}{\sqrt{1-v^2/c^2}} \qquad (1.1)$$
$$y'=y, \quad z'=z$$

do not change the interval s^2, i.e. the following identity holds

$$s^2\equiv c^2t^2-\mathbf{r}^2=c^2t'^2-\mathbf{r}'^2\equiv s'^2 \qquad (1.2)$$

2. Using Eqs.(1.1) find relation between the period of time Δt, which some event takes in the reference frame K and a period of time $\Delta t'$ for this event in K'.
   Hint. Consider K as 'earth' and K' as a 'spacecraft'. When an observer on earth watches for how long something happens on the spacecraft, he/she takes $\Delta x'=0$.
3. Consider a 4-vector a^μ, $\mu=0,\ldots,3$. Write down relations between components of this vector in the systems K and K'.
4. Consider the 4-acceleration of a particle w^μ, which is defined via the vector of 4-velocity u^μ

$$w^\mu=\frac{du^\mu}{ds}$$
$$u^\mu=\frac{dx^\mu}{ds} \qquad (1.3)$$

Prove that the 4-acceleration satisfies an identity

$$w^\mu w_\mu=-\frac{\mathbf{a}^2}{c^4} \qquad (1.4)$$

where $\mathbf{a}=\ddot{\mathbf{r}}$ is a conventional acceleration (3-vector) in the rest frame, in which the particle is at rest at a given moment of time.

5. Write relations, which define the field $F_{\mu\nu}$ via the potential A_ν. Prove that

[1] You welcome, if you wish, to use here and below the system of units in which c=1.

$$F_{\mu\nu}=\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix} \tag{1.5}$$

6. Define gauge transformations for the potential A_μ. Verify that the field $F_{\mu\nu}$ is invariant under gauge transformation

II. **Radiation** (25% mark)

1. Derive the formula

$$\Delta\mathrm{E}=\frac{2e^4}{3m^2c^3}\int_{-\infty}^{\infty}\frac{(\mathbf{E}+\mathbf{v}\times\mathbf{B}/c)^2-(\mathbf{v}\cdot\mathbf{E})^2/c^2}{1-v^2/c^2}dt \tag{2.1}$$

for the total energy $\Delta\mathrm{E}$ radiated by a relativistic particle which propagates in an external electric field **E**.

Hints:

a. Remember that in the reference frame, in which the particle is at rest at a given moment of time the radiated energy dE and momentum $d\mathbf{P}$ satisfy conventional dipole rules

$$dE=\frac{2e^2}{3c^3}a^2dt \qquad d\mathbf{P}=0 \tag{2.2}$$

Here e is the charge of the particle, and a is the acceleration of the particle in the rest frame.

b. Show that in any other reference frame the radiated 4-momentum $dP^\mu=(dE/c,d\mathbf{P})$ satisfies

$$dP^\mu=-\frac{2e^2}{3c}w^\nu w_\nu dx^\mu \tag{2.3}$$

Here w^ν is the 4-acceleration defined in Eq.(1.3).

c. Use the equations of motion

$$mc\frac{du^\nu}{ds}=eF^{\nu\kappa}u_\kappa \tag{2.4}$$

to express 4-accelerations in Eq.(2.3) via the fields and 4-velocities.

d. Use Eq.(1.5).

2. Consider a charged particle, which accelerates from rest by a static homogeneous electric field $\boldsymbol{E}$.
   a. Find the energy $\Delta\mathrm{E}$ radiated by this particle during a period of time T (from 0 to T).

b. Compare this energy with the total energy, which the particle acquires from the field during same period of time.
c. Find restriction on the field $\boldsymbol{E}$, which guarantees that the radiated energy is much smaller than the total energy of the particle. How well this restriction is satisfied in conventional processes in laboratory, in atoms, in nuclei *etc* ?

Part II

Question 3. Phenomenology. (25%)

a. Name all known leptons and quarks. How many generations are known ?
b. Present values for the spins, $SU(2)$ isotopic spins, and electric charges of leptons and quarks in the first generation.
c. Name all known gauge bosons.
    - Indicate their spins
    - Indicate values for their masses (approximately)
    - Indicate an ability of each gauge boson to propagate as a free particle

Consider a naïve, simplified physical picture for hadrons, in which they are constructed from a minimal possible number of quarks ("Lego-type" model).

d. How many quarks are necessary to build the neutron n ? Name them.
e. How many quarks are necessary to construct a pion π^- ? Name them.
   Hint: remember that the neutron is a fermion and pion is a boson, and that they are relatively light particles.
f. Draw simplest Feynman diagrams, which describe elastic scattering of electron and positron with parity violation
$$e^+ + e^- \to e^+ + e^- \tag{3.1}$$
g. Draw Feynman diagrams, which describe same process of electron and positron scattering, in which parity is conserved.

Question 4. Standard Model. (25%)

a. Explain how the Higgs mechanism generates mass for photons in superconductors.
   Hints.
    - Explain firstly that the superconducting current can be written as
$$\vec{j} = \frac{n_c e_c}{m_c}\left(\vec{\nabla}\phi - e_c\vec{A}\right) \tag{4.1}$$

where n_c, e_c, m_c are the density of the condensate, and charge and mass of the Cooper's pair.

- Use the Maxwell's equations to derive the equation governing the magnetic field. (Take a curl in the Ampere's law.)
- Substituting in this equation a plain-wave solution
$$\vec{B} = \vec{B}_0 \exp\left[i\left(kx - \omega t\right)\right] \tag{4.2}$$
find dispersion relation, i.e. relation between k and ω.
- Find minimal ω.

Consider the Weinberg-Salam $U(1)\times SU(2)$ of electro-weak interactions in which a scalar field (Higgs field) is transformed as a doublet under $SU(2)$ (isospin $1/2$) and is also transformed nontrivially under $U(1)$. Presume that the Higgs field develops the vacuum expectation value

$$\varphi_{\text{vac}} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{4.3}$$

b. Explain fundamental physical reasons, which allow to transform this value to a canonical form

$$\varphi_{\text{vac}} = \begin{pmatrix} 0 \\ \text{v} \end{pmatrix}, \ \text{v} > 0 \tag{4.4}$$

c. Describe gauge transformations from the $U(1)\times SU(2)$ gauge group, which leave the vacuum expectation value of the scalar filed (4.4) unchanged.

   i. How many parameters govern this family of gauge transformations.

   ii. Prove that if U_1 and U_2 belong to this family (i. e. they are gauge transformations, which leave the vacuum expectation value of the scalar field unchanged), then these transformations commute. This means that if one makes firstly one of the two gauge transformations and then fulfils another one, then the resulting gauge transformation does not depend on which one of the two transformations goes first and which one second.

   Hint: this is a one-liner (literally).

d. Argue *briefly* why the photon state should be constructed as a linear combination of the gauge boson B_μ, which describes the original $U(1)$, and the W_μ^3-boson, which is one of the gauge bosons of the $SU(2)$

$$A_\mu = \cos\theta \ B_\mu + \sin\theta \ W_\mu^3 \tag{4.5}$$

e. Consider a term in the SU(2) Lagrangian, which describes interactions of leptons (first generation) with gauge bosons

$$L_{\text{int}}^{SU(2)} = \frac{g_2}{2}\left(\bar{L}\gamma^{\mu} W_{\mu}^{a} \tau^{a} L\right) \tag{4.6}$$

where L is a doublet

$$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \tag{4.7}$$

Extract from the Lagrangian (4.6) those terms, which are responsible for interaction of leptons with ~~charged~~ bosons Z_{μ}. Remember that ✓

$$Z_{\mu} = W_{\mu}^{3} \tag{4.8}$$

and the Pauli matrixes in Eq.(4.6) read

$$\tau^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4.9}$$

Hint: the necessary terms in the Lagrangian are proportional to either the current $\left(\bar{e}_L \gamma_{\mu} e_L\right)$ or the current $\left(\bar{\nu}_e \gamma_{\mu} \nu_e\right)$.