

THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS 4103 – 2011

Solid State Physics

1. Time Allowed: 3 hours
2. Total number of questions: 4
3. Marks available for each question are shown in the examination paper. The total number of marks is 40.
4. Attempt ALL questions!
5. Question 1+2 and 3+4 must be answered in a separate book.
6. Candidates may bring their own calculators (calculators without alphabetic keyboards).
7. Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work. Do not use red ink.
8. The paper may be retained by the candidate.

Note that the absorption coefficient is given by:

$$\begin{aligned}\alpha(\omega) &= \frac{2\pi e^2 \omega}{\epsilon_0 c_0 n} \int_{\text{BZ}} \frac{d\vec{k}}{4\pi^3} |M_{CV}|^2 \delta(E_c(\vec{k}) - E_v(\vec{k}) - \hbar\omega) \\ &= \frac{2\pi e^2 \omega}{\epsilon_0 c_0 n} |M_{CV}|^2 J_{CV}(\hbar\omega)\end{aligned}$$

where c_0 is the speed of light and n is the index of refraction of the material. J_{CV} is the joint density of states.

The imaginary part of the dielectric function of a material is proportional to the absorption coefficient:

$$\alpha = \frac{\omega}{c_0} \frac{\epsilon_i}{n} \quad \epsilon_i(\hbar\omega) = \frac{2\pi e^2}{\epsilon_0} |M_{CV}|^2 J_{CV}(\hbar\omega)$$

(a) Why are optical transitions, i.e. transitions which can be observed by spectroscopies using visible light, only vertical transitions in the band structure.

(b) What is the difference between a direct and an indirect semiconductor and how does this influence the the electronic transition of a photoluminescence experiment.

(c) Photoluminescence on a direct semiconductor such as GaAs:

(i) Briefly sketch the experimental setup of a photoluminescence experiment.

(ii) Explain all electronic transitions involved in a photoluminescence process.

(iii) How can the intensity, i.e. transition probability, of the emitted photoluminescence light be derived? Explain all relevant terms of the equation.

(d) Explain the terms 'Van-Hove Singularity' and 'Joint Density of States' briefly. Where do the Van Hove Singularities appear in the above given band-structure of Silicon.

(e) The imaginary part of the dielectric function can be measured by optical ellipsometry and is proportional to the absorption coefficient. Describe the peaks in the above given imaginary part of the dielectric function ϵ_i in the context of Van Hove singularities.

Question 1 (10 marks):

(a) Starting from the discrete electronic levels of well separated free electrons, describe how the electronic bands in a solid are formed when the distance between the atoms is reduced.

(b) Construction of the Hamilton operator of a solid: Explain the various terms which contribute to the Hamilton operator of a solid. Which additional term has to be included if the material is ferromagnetic?

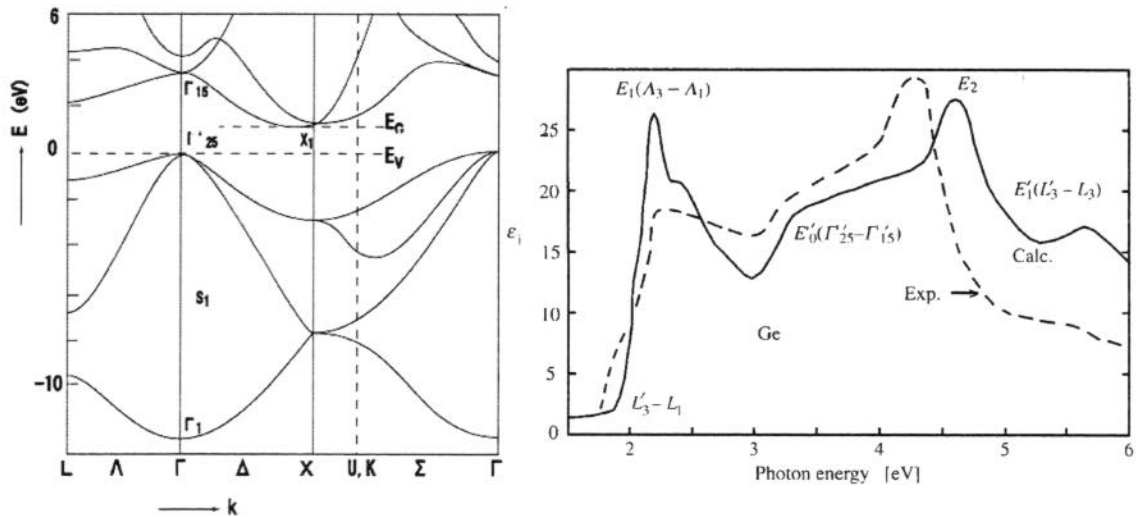
(c) Why is it problematic to solve the Schrödinger equation of a solid directly? Describe the various approximations, which are required to solve the Schrödinger equation of the electronic system of a solid.

(d) The electronic band-structure of a solid like Silicon (see below) can be calculated in the 'Weak Binding Limit'. Explain this technique briefly.

(e) Explain why and how the electronic band-gap of a semiconductor changes with increasing temperature due to the lattice expansion.

Question 2 (10 marks):

The left part of the figure given below shows the band structure of Silicon and the right side displays the imaginary part of the dielectric function of Germanium (dielectric function: $\epsilon(\hbar\omega) = \epsilon_r(\hbar\omega) + i\epsilon_i(\hbar\omega)$).



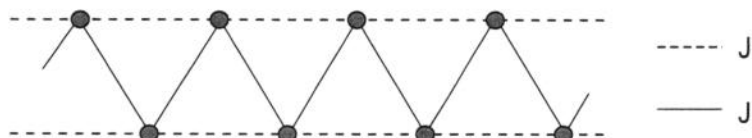
Question 4 (10 marks):

(a) The Heisenberg model provides a microscopic approach to the physics of magnetic materials with localized moments.

(i) Write down a Hamiltonian for this model and explain the meaning of all terms and symbols.

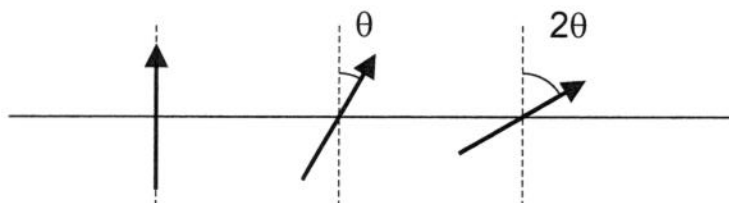
(ii) In 1-2 pages explain what are spin-waves or 'magnons'. In your answer you should compare and contrast ferromagnets and antiferromagnets, discuss the form of magnon energy spectra (dispersion curves), and the consequence for low temperature thermodynamics. How can magnons be observed and their energies measured?

(b) In a certain class of magnetic materials, the magnetic ions form 1-dimensional structures as shown:



where the nearest neighbour interactions J are ferromagnetic and the next-nearest neighbour interactions J' are antiferromagnetic. The interactions between the chains are weak and can be neglected.

Consider a possible ordered state in which successive spins are canted by an angle, as shown below (all spins are in the plane of the figure):



(i) Assuming 'classical' vector spins of length S , write down an expression for the energy per site, in terms of J, J', θ .

(ii) Minimize the expression with respect to θ and briefly discuss the two types of solutions.

(c)(i) Briefly explain the 'rigid-band' model for $3d$ transition metals and show how this can explain the occurrence of a non-integral moment per site.

(ii) Fe and Ni have, respectively, 8 and 10 valence electrons to be distributed over the $3d$ and $4s$ bands. Transport measurements show that the effective $4s$ electron configurations are $4s^{0.94}$ for Fe and $4s^{0.6}$ for Ni, and the atomic moments at low temperature are $2.2\mu_B$ for Fe and $0.6\mu_B$ for Ni. Calculate the occupancies for the $3d$ 'spin-up' and 'spin-down' bands for Fe and Ni on the basis of the 'rigid band' model.

Question 3 (10 marks):

(a) Insulating magnetic materials usually contain ions from two regions of the periodic table. Identify these and briefly discuss similarities and differences between these two classes of magnets.

(b) A simple approach to describe the physics of magnetically ordered systems is via the 'mean-field' or 'molecular field' approach (MFA). Explain the basic idea of the MFA (1-2 paragraphs, no mathematics) and discuss its successes and failures.

(c) For a spin $S = 1/2$ ferromagnet MFA leads to the following equation for the dimensionless magnetic moment per ion,

$$m \equiv M/g\mu_B S$$

$$m = \tanh(am + bB)$$

where, $a = zJS^2/k_B T$, $b = g\mu_B S/k_B T$, with B the applied field, and the other symbols have the usual meaning.

(i) Show how this equation can be solved graphically in zero applied field and obtain an expression for the Curie temperature T_C in terms of the parameters given.

(ii) Show that, as $T \rightarrow T_C^-$, the magnetization behaves as:

$$m \sim \text{const.} \left(1 - \frac{T}{T_C}\right)^{1/2}$$

(iii) Show that, for $T > T_C$, the zero-field susceptibility has the Curie-Weiss form:

$$\chi \sim \frac{\text{const.}}{T - T_C}$$

(iv) Estimate the size of the molecular field in iron ($T_C \sim 1000$ K). (order of magnitude only).

The following may be needed:

$$\tanh x = x - \frac{1}{3}x^3 + \dots$$

$$k_B = 8.617 \cdot 10^{-5} \text{ eV/K}, \quad \mu_B = 5.788 \cdot 10^{-5} \text{ eV/T}$$