Part 2. Standard Model.

Question 3 (25 marks)

a) List the known fundamental forces in nature. For each force, name the boson(s) which carry or mediate this force, and indicate whether they are massive or massless. Also indicate whether each force is short-ranged or long-ranged.

b) How many generations of quarks and leptons are known? List the leptons and quarks in each generation. Which forces are experienced by the lepton family?

c) Draw a Feynman diagram describing the process $e^- p \to \nu_e n$. What is the fundamental interaction involved here?

d) The interaction Lagrangian of the electroweak theory involving the (u, d) quarks is

$$L_{int} = \frac{g_1}{2}\{Y_L(\bar{L}\gamma^\mu L) + Y_R^u(\bar{u}_R\gamma^\mu u_R) + Y_R^d(\bar{d}_R\gamma^\mu d_R)\}B_\mu + \frac{g_2}{2}(\bar{L}\gamma^\mu\tau^i L)W_\mu^i \quad (1)$$

where L is the weak isotopic doublet $L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$

and u_R, d_R are weak isotopic singlets. We know that

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu \quad (2)$$

$$W_\mu^3 = W_\mu^0 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu \quad (3)$$

in terms of the physical photon and Z-boson fields, A_μ and Z_μ respectively; and that

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad \cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}$$

while

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The electric charges of the u and d quarks are $+2e/3$ and $-e/3$, respectively. By examining the neutral current terms, determine the values of the weak "hypercharges" Y_L, Y_R^u, Y_R^d for the up and down quarks.

Question 4 (25 marks)

a) Briefly discuss in words the role played by the mechanism of spontaneous symmetry-breaking in the standard model.

b) A complex scalar field has Lagrangian density

$$L = (\partial_\mu\phi^\dagger)(\partial^\mu\phi) - \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 \quad (4)$$

Show that L is invariant under a global U(1) gauge transformation

$$\phi \to \phi' = e^{i\chi}\phi \quad (5)$$

c) Re-express L in terms of the real scalar fields ϕ_1, ϕ_2 where

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \tag{6}$$

d) Derive an expression for the Hamiltonian (energy) density of the system

$$H = \sum_i \dot{\phi}_i \frac{\partial L}{\partial \dot{\phi}_i} - L \tag{7}$$

Assume $\mu^2 < 0$, and give a 3D sketch of the potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \tag{8}$$

as a function of (ϕ_1, ϕ_2). Hence specify the (degenerate) classical ground state of this system.

e) Assume the vacuum expectation value of the fields are

$$< \phi_1 >_0 = v, \quad < \phi_2 >_0 = 0$$

where $v = \sqrt{-\mu^2/\lambda}$. Let

$$\phi_1(x) = v + \eta(x), \quad \phi_2(x) = \rho(x)$$

and re-express the Lagrangian in terms of fields $\eta(x)$, $\rho(x)$. Expand L up to terms of second order in the fields. By inspection of the result, give the masses of the elementary excitations in the model. What is the significance of this result?

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