QUESTION 1. (7.5 marks)

a) Show that the transformation

$$Q = p + iaq \qquad \text{and} \qquad P = \frac{p - iaq}{2ia}$$

is a canonical.

b) Find the generating function of type $F_2(q,P)$ for the transformation where

$$p = \frac{\partial F_2(q,P)}{\partial q} \qquad \text{and} \qquad Q = \frac{\partial F_2(q,P)}{\partial P}.$$

c) If the Hamiltonian is $H = \frac{1}{2}\left(p^2 + a^2 q^2\right)$, find the new Hamiltonian $K(Q,P)$ generated by the transformation.

d) Use Hamilton's equations of motion to find the equations of motion for $K(Q,P)$.

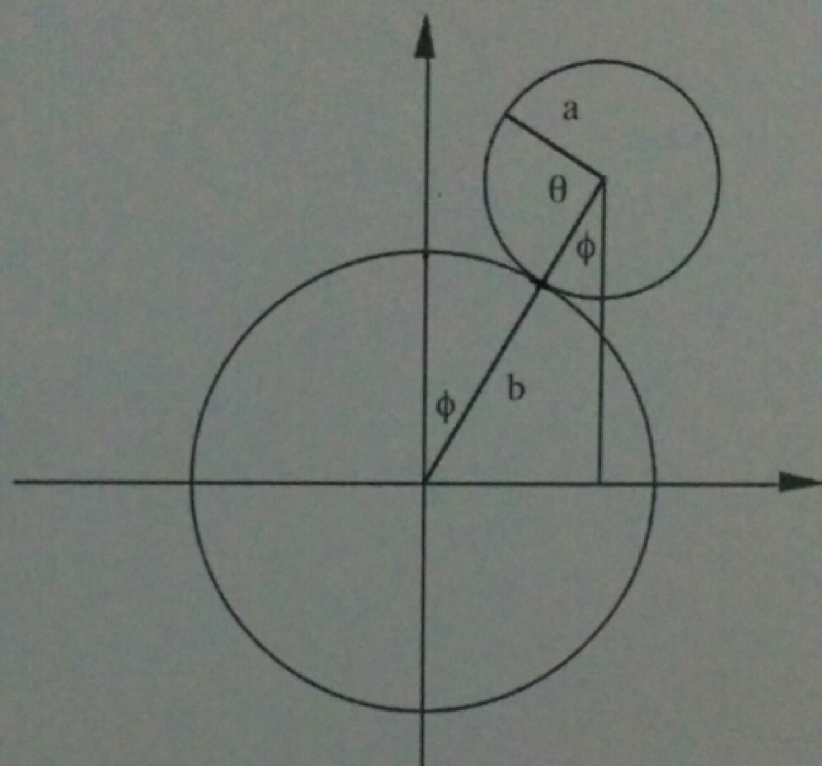
e) Solve these equations of motion to find the time evolution of Q and P.

f) If u is a function of q,p,t write down the Poisson bracket form for the equation of motion for u.

g) Hence derive the Poisson bracket equations of motion for q and p.

QUESTION 2. (7.5 Marks)

A sphere of radius a and mass m rests on top of a fixed sphere of radius b. The first sphere is slightly displaced so that it rolls without slipping down the second sphere. Where will the first sphere leave the second sphere?



(a) Determine the kinetic and potential energy of the moving sphere in polar coordinates and find the Lagrangian for this system.

(b) Identify the constraints and find the equations of constraint.

(c) Use the method of Lagrange's undetermined multipliers **to obtain the equations of motion** for the system in spherical polar coordinates.

(d) Show that eliminating the no slipping constraint gives the following equation for ϕ

$$\tfrac{7}{5}x\ddot{\phi} - g\sin\phi = 0$$

(e) Determine the initial conditions for $\dot{\phi}(\phi)$.

(f) If $\lambda_1 = mg\cos\phi - mx\dot{\phi}^2$, solve for λ_1.

(g) Explain the physical meaning of the multiplier λ_1.