PHYS3230/3011 Electrodynamics 2015: Mid-session test

Thursday April 16, 2015, 5–6pm

Question 1

Consider the scalar and vector potentials

$$\phi_X(\mathbf{r},t) = 0, \quad \mathbf{A}_X(\mathbf{r},t) = -\frac{1}{4\pi\epsilon_0}\frac{qt^2}{r^2}\hat{\mathbf{r}},$$

in some gauge X.

(a) Find the **E** and **B** fields.

(b) Find the corresponding charge and current distributions.

(c) Find a gauge function $\lambda(\mathbf{r},t)$ which transforms ϕ_X and $\mathbf{A}_X$ to the Coulomb gauge.

(d) Compute the scalar potential ϕ_C in the Coulomb gauge.

Hints: $\nabla\cdot(\hat{\mathbf{r}}/r^2) = 4\pi\delta^{(3)}(\mathbf{r})$, and $\nabla(1/r) = -\hat{\mathbf{r}}/r^2$.

Question 2

Consider an elliptically polarised plane wave of angular frequency ω and wavenumber k propagating in free space in the z-direction.

(a) Write down the expression for this plane wave.

(b) Find the corresponding magnetic field and Poynting vector.

(c) Calculate the intensity of this wave crossing a surface parallel to the xy-plane.

(d) Suppose the wave strikes, at right angle, the surface of a medium with permittivity ϵ and permeability μ. How much energy is transmitted across the surface per unit area per unit time?

Hint: $\int_0^T dt\,\cos^2(a-t) = \int_0^T dt\,\sin^2(a-t) = T/2$.