Electromagnetism

Phys 3230, mid-session exam 15-04-2011

All questions should be addressed. If one is not certain in maths, one should try to present explanations in words. Calculators are not forbidden, though unlikely they could be useful.

I. Maxwell's equations (33.3%)

a. Write down Maxwell's equations in conventional differential form.

b. Write down gauge transformations for the potentials V and **A**. Prove that the electrodynamics is a gauge invariant theory.

c. Write down the current conservation law in differential form and verify that Maxwell's equations comply with it. Use the Gauss theorem to present the current conservation law in an integral form.

d. Consider a volume $\mathcal{V}$ surrounded by the surface $\mathcal{A}$ assuming that $\mathcal{V}$ and $\mathcal{A}$ are time-dependent so that as a function of time the volume $\mathcal{V}$ can change its size and shape.

- Suggest the integral form for the current conservation law, which one could think is valid for this case (speculate, do not derive).
- Explain briefly, qualitatively the difference of the equation you suggest from the traditional form for the current conservation law in which the volume and surface are taken as static (as it was assumed in Q1c).

II. Simple applications (33.3%)

a. Consider an infinitely long tube of radius R. Presume that its surface is homogeneously charged with the charge density σ (C/m^2), and that it is empty inside. Calculate the electric and magnetic fields **E** and **B** created by this device if they are studied by a (very clever and able) sparrow, which flies along the tube axis with the velocity v. Find **E** and **B**, which the sparrow would measure, if it travels

- inside the tube, $\rho < R$
- outside it, $\rho > R$

III. Electromagnetic waves (33.3%)

a. Consider a plane, monochromatic, circularly polarized electromagnetic wave, which propagates along the z-direction, has frequency ω and electric filed E.

- Calculate the density of energy u_{em} and Poynting vector **S** for this wave.

(*Continued on p.2*)

- Verify by explicit calculations that the found u_{em} and $\mathbf{S}$ comply with the energy conservation law. (Hint: do not get confused if the necessary calculations prove simple.)