UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2011

PHYS3230

Electromagnetism

Time Allowed – 2 hours

Total number of questions - 4

Answer ALL questions

All questions are of equal value

Candidates must supply their own, university approved, calculator.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Candidates may keep this paper.

Electromagnetism Phys 3230

Exam 2011

All questions should be addressed. If one is not certain in maths, one should try to present explanations in words.
Use the conventional SI units; alternatively one is welcomed to use units in which $\varepsilon_0 = \mu_0 = c = 1$ indicating then clearly that this specific system is applied.

1. **Maxwell's equations (25 %)**
   a. Write down Maxwell's equations in conventional differential form.
   b. Using the Gauss' and Stoke's theorems present them in the integral form.
   c. Explain briefly the difference between the physical phenomena described by the derivatives $\frac{\partial}{\partial t}\Phi_E$ and $\frac{d}{dt}\Phi_E$. Present very briefly, qualitatively the arguments, which favour the presence of $\frac{d}{dt}\Phi_E$ in the Faraday law.
   d. Express the electric and magnetic fields via the potentials V and **A.** Verify that the first pair of Maxwell's equations (equations, which do not include the charge and current) are satisfied for arbitrary potentials.
   e. Derive the differential equation, which relates the potential **A** with the charge and current densities $\rho, \mathbf{j}$ using the gauge condition V= 0.

   Hint: it is useful to rely on the identity

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla (\nabla \cdot \mathbf{a}) - \Delta \mathbf{a} \tag{1.2}$$

2. **Conservations laws (25 %)**
   a. Explain the physical meaning of the quantities

$$u_{em} = \frac{\varepsilon_0 \mathbf{E}^2}{2} + \frac{\mathbf{B}^2}{2\mu_0}$$

$$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \tag{2.1}$$

$$T_{ij} = \varepsilon_0 \left(E_i E_j - \frac{1}{2} E^2 \delta_{ij} \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

   b. Write down the energy conservation law for the system of charged particles and electromagnetic field in differential and integral forms.
   c. Derive the momentum conservation law for the electromagnetic field in the vacuum (in the volume, where no charged particles are present).

   Hint: the derivation needs the following identity

$$\left[\,\mathbf{a}\times(\nabla\times\mathbf{a})\,\right]_i = \frac{1}{2}\nabla_i(\mathbf{a}^2) - \sum_{j=1}^{3}\nabla_j(\mathbf{a}_j\mathbf{a}_i) + \mathbf{a}_i(\nabla\cdot\mathbf{a}) \tag{2.2}$$

Here i,j = 1,2,3 mark x,y,z components of vectors.

d. Consider static case. Express the force applied to the charged particles located inside the given volume via the stress tensor on the surface surrounding this volume.

3. **Maxwell's equations in matter (25 %)**

a. **Metals**. Present Maxwell's equations in metals in terms of **E** and **B** presuming that

- Ohm's law is valid (write down the Ohm law for metals in terms of the current density $\mathbf{j}$, electric field **E**, and conductivity σ),
- there are no external charges, $\nabla\cdot\mathbf{E}=0$,
- lattice polarization is negligible.

b. From Maxwell's equations (considered in Q3a) derive the equation, which governs propagation of the electric field **E** in metals.

c. Consider propagation of the monochromatic wave with frequency ω in metals.

- Indicate the region of frequencies in which the metal is called the good metal
- explain what is called the skin depth and derive the expression for it for good metals

d. **Superconductors.** Present (very briefly) simple, intuitive, physical arguments, which justify validity of the London equations

$$\frac{\partial}{\partial t}\mathbf{j} = \frac{ne^2}{m}\mathbf{E}$$
$$\nabla\times\mathbf{j} = -\frac{ne^2}{m}\mathbf{B} \tag{3.1}$$

Explain what n, e and m are.

e. Derive the expression for the London penetration depth λ, which measures how deeply the magnetic field penetrates into the superconductor. Explain how λ depends on n, e and m.

4. **Electromagnetic waves (25 %)**

a. **Wave propagation**. Consider the electric filed

$$\mathbf{E}(z,t) = E_0\, f(z-ct)\,\mathbf{e}_x \tag{4.1}$$

where *f(w)* is the given (arbitrary) function and E_0 is a constant.

- prove that this field satisfies the wave equation
- find such magnetic field $\mathbf{B}(z,t)$ that the pair of the fields $\mathbf{E}(z,t)$ and $\mathbf{B}(z,t)$ satisfies Maxwell's equations.

  Hint: keep in mind the conventional geometrical relations between the direction of propagation of the electromagnetic wave **n**, fields **E** and **B**.
- find the density of energy u_{em}, the Pointing vector $\mathbf{S}(z,t)$ and Maxwell's stress tensor $T_{ij}(z,t)$ for the wave, which electric field satisfies Eq.(4.1).
- write down the function *f(w)*, which describes the monochromatic plane wave propagating with a given frequency.

b. **Scattering**. Consider a molecule, which is placed in the field of the monochromatic plane electromagnetic wave propagating along the z-axis and having polarization along the x-axis, the amplitude of electric field E_0, and the frequency ω. Assume that its wavelength is much larger than the size of a molecule, and frequency is lower than frequencies related to molecular excitations.
Assume also for simplicity that the molecular polarizablity α is frequency independent.
Hint: remember that the polarizablity is a coefficient in the relation between the molecular dipole moment and electric field, for the frequency independent polarizablity it reads

$$\mathbf{d}(t) = \alpha\,\mathbf{E}(t) \tag{4.2}$$

- find the "averaged" intensity (averaged over time) of the dipole radiation I produced by this molecule (from this point on there are two electromagnetic waves. The initial wave and the secondary one, the scattered wave, emitted by the atom)
- explain where the angular distribution of the scattered radiation has its maximum, and in which direction it is minimal.
- the scattering of light by atoms and molecules is responsible for the light produced by the sky. Explain the physical origin of the blue colour of our sky.
- explain why and how the light from the sky is polarized.
- find the ratio of the "averaged" intensity of the scattered radiation I to the flux of energy S_0 in the original electromagnetic wave (also averaged over time). This important quantity is called the cross section σ for scattering

$$\sigma = \frac{I}{S_0} \tag{4.3}$$