# Electromagnetism Phys 3230, mid-session 23/04/2010

All questions should be addressed. If one is not certain in maths, one should try to present explanations in words. Calculators are not forbidden, though would not be useful.

1. **Maxwell's equations (50 %)**
   a. Write down Maxwell's equations in the vacuum in the conventional differential form.
   b. Using Gauss' and Stoke's theorems rewrite the Maxwell's equations in the integral form.
   c. Express the electric **E** and magnetic **B** fields via the potentials $V$ and **A.**
   d. Describe gauge transformations for the potentials. Explain what is called gauge invariance. Prove that the electrodynamics is gauge invariant.

2. **Applications (50 %)**
   a. Using Maxwell's equations prove that the magnetic field in the vacuum satisfies the wave equation

$$\partial^2 \mathbf{B} \equiv \left( \frac{1}{c^2} \frac{\partial^2}{\partial t} - \Delta \right) \mathbf{B} = 0 \qquad (2.1)$$

   Hint: remember that

$$\nabla \times (\nabla \times \mathbf{a}) = \nabla(\nabla \cdot \mathbf{a}) - \Delta \mathbf{a} \qquad (2.2)$$

   b. Suppose that the magnetic field depends only on two variables $x$ and $t$, $\mathbf{B} = \mathbf{B}(x,t)$. Write down the most general functional form for the magnetic field, which is allowed by Eq.(2.1) (disregarding any additional restrictions, which may stem from Maxwell's equations).
   c. Consider two large parallel metal plates separated by a small distance $d$. Assume that over each plate runs a current; currents on each plate have the same magnitude, same direction along the $z$-axis, and both are homogeneously spread over the plate's surface with the density $dI/dx = \Upsilon$, see Fig.1.
      - Find the magnetic field between the plates and outside.
      - Find the absolute value and direction of the pressure $p$ (force per unit area) on each plate.

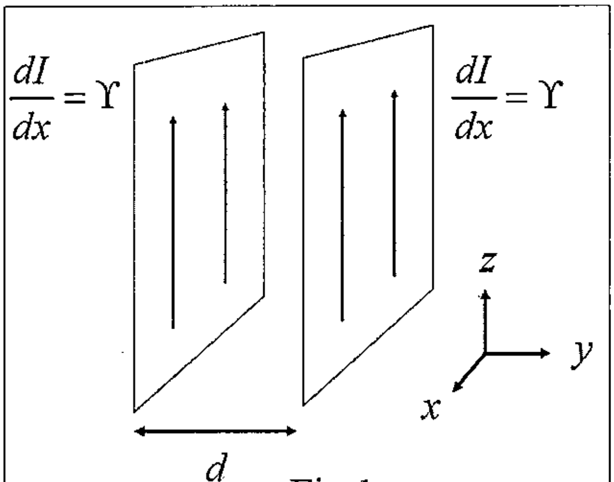

Fig.1