Quantum Mechanics – PHYS 3210

THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Mid-Session test April 2014

Time Allowed – 1 hour

Total number of questions – 3

ALL questions need to be addressed

Question marks range from 30 to 35.

This paper may be retained by the candidate.

Students may provide their own UNSW approved calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Question 1. Heisenberg's uncertainty principle (Marks 30).

a. Formulate the Heisenberg uncertainty principle (very briefly).

b. Assume that a particle of mass m propagates in the potential

$$U(r) = -\frac{Z\,e^2}{r} \qquad (1)$$

where Z>0 is a positive constant (charge). Using Heisenberg's uncertainty principle, estimate the ground state energy E, as well as the averaged kinetic and potential energies in this state.

Hint. To simplify calculations one can choose units $\hbar = m = e = 1$, but the final answer should be presented in conventional units. If you would struggle with algebra, keep in mind that up to a numerical factor the dependence of the energy on Z, $\hbar$, e and m can be recovered from simple dimensional analyses.

Question 2. Quantum oscillator (Marks 35).

Consider the conventional quantum oscillator described by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}, \quad \hat{p} = -i\hbar\frac{d}{dx} \qquad (2)$$

a. Verify that the wave function

$$\psi(x) = \frac{1}{\pi^{1/4}\sqrt{b}}\exp\left(-\frac{x^2}{2b^2}\right), \quad b = \sqrt{\hbar/m\,\omega}, \qquad (3)$$

$\frac{1}{2}\hbar\omega$?

satisfies the stationary Schrodinger equation. Find the corresponding energy. Prove that this wave function describes the ground state energy. Here and below it may be convenient to choose units $\hbar = \omega = m = 1$ for intermediate calculations.

b. Consider a slightly modified 3D harmonic oscillator described by the Hamiltonian

$$\hat{H}_{3D} = \frac{1}{2m}\hat{\boldsymbol{p}}^2 + \frac{m\omega^2}{2}r^2 + V\hat{p}_x - Fy, \qquad (4)$$

where $\boldsymbol{r}$ is the radius vector and $\hat{\boldsymbol{p}} = -i\hbar\nabla$ is the corresponding momentum, while V, F are real-valued constants. Find the energy spectrum of $\hat{H}_{3D}$ (all energy levels). In addition, present explicitly the wave function of the ground state.

Question 3. Semiclassical approximation (Marks 35).

I think...

a. Consider the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(x), \quad \hat{p} = -i\hbar\frac{d}{dx} \qquad (5)$$

and assume that $U(x)$ is a sufficiently smooth potential. Using the semiclassical approximation, write down the wave function for the energy level E.

b. Consider the Hamiltonian (2). Verify that at large x, $x \gg b$, the semiclassical approximation $\psi_{SC}(x)$ for the wave function complies with the exact one, which is presented in Eq.(3), i.e. prove that $\psi_{SC}(x) \approx \psi(x)$.

For simplicity restrict your discussion to the exponential accuracy, i.e. keep attention to what is happening in the exponent of the wave function and disregard the pre-exponential factor.