THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS
FINAL EXAMINATION
JUNE 2011

PHYS3210
Quantum Mechanics

Time allowed – 2 hours
Total number of questions – 4
Answer ALL 4 questions
All questions are of equal value
Candidates must supply their own,
university-approved calculator.
Answers must be written in ink.
Except where they are expressly required,
pencils may only be used for drawing,
sketching, or graphical work.
Candidates may keep this question paper.

Question 1

A hydrogen atom is in a state described by the following wavefunction at time $t = 0$:

$$\Psi(\underline{r}, 0) = \frac{\sqrt{3}}{2}\,\psi_{100} \;+\; \frac{1}{2}\,\psi_{211}$$

(a) What is the expectation value of the energy at time $t = 0$? (Express your answer in eV.)

(b) What is the wavefunction $\Psi(\underline{r}, t)$ at time t ?

(c) What is the expectation value of the energy at time t?

(d) What is the probability of finding the atom in its ground state ?

(e) What is the probability of finding the atom in an $\ell = 1$ state ?

(f) Evaluate the probability function $|\Psi(\underline{r}, t)|^2$ and find its time dependence.

NB (1) You do not need to know the explicit form of the energy eigenfunctions: you should express your answers to parts (b) and (f) in terms of ψ_{100} and ψ_{211}.

(2) The energy levels of the hydrogen atom are given by $E_n = \dfrac{-13.6\ eV}{n^2}$

Question 2

(a) Given two wavefunctions ψ and ψ' which are both linear superpositions of two orthonormal wavefunctions ψ_1 and ψ_2

ie $\psi = a\psi_1 + b\psi_2 \qquad (a^2 + b^2 = 1)$

and $\psi' = c\psi_1 + d\psi_2 \qquad (c^2 + d^2 = 1)$

find one more condition relating a, b, c, d if ψ and ψ' are also orthonormal.

(b) Two electrons are prepared with their total spin projection along the z-axis of zero. Show that there are two wavefunctions (in terms of the spin projections of the individual electrons) describing this situation.

(c) Normalise these wavefunctions, and use the result of part (a) to show that they are orthogonal.

P.T.O.

Question 3

A particle in a p-state is represented by the wavefunction $\psi = 4\,Y_1^1 \; + \; 3\,Y_1^{-1}$

where Y_ℓ^m are the (normalised) spherical harmonics for quantum numbers ℓ and m.

(a) Normalise ψ.

(b) What is the probability that a measurement of L_z will give the value $+\hbar$?

(c) What is the expectation value of L_z ?

(d) What is the expectation value of L_z^2 ?

(e) What is the expectation value of L^2 ?

(f) What are the possible values of a measurement of L_x ?

(g) What is the expectation value of L_x^2 ?

Question 4

A particle of mass m and charge e is confined to the one-dimensional infinite potential well given by:

$V(x) = 0$ if $0 \le x \le a$, $\qquad V(x) = \infty$ if $x < 0$ or $x > a$.

The normalised eigenfunctions for this system are: $\psi_n(x) = \dfrac{\sqrt{2}}{a} \sin \dfrac{n\pi x}{a}$

(a) A small perturbation to the potential is applied at the mid-point, $x = a/2$.
The product of the width and height of this perturbation is described by a Dirac delta-function, ie $v(x) = A\,\delta(x = a/2)$.
Find the first-order corrections to the energies of the first three levels.

The above perturbation is removed, and the system is now perturbed by a uniform electric field E applied in the positive x direction.

(b) Write down the new perturbing potential $v(x)$.

(c) Show that the first-order corrections to all the energy levels have equal value.

NB $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

University of New South Wales – School of Physics

PHYS3210 – Quantum Mechanics – Final Exam, 14 June 2011

Corrections and Clarifications
(To be read in conjunction with the main exam paper)

Question 2

(a) ...

(b) Two electrons are prepared with their total spin projection along the z axis, $S_z = 0$.
Write down the two wavefunctions which are eigenstates of the particle-exchange operator (these are also eigenstates of the total spin, S), in terms of the spin-up and spin-down states of the two electrons.

(c) ...

Question 4

A particle of mass m and charge e is confined to the one-dimensional infinite potential well given by:

$V(x) = 0$ if $0 \leq x \leq a$, $\qquad V(x) = \infty$ if $x < 0$ or $x > a$.

The normalised eigenfunctions for this system are: $\psi_n(x) = \dfrac{\sqrt{2}}{a} \sin \dfrac{n\pi x}{a} \qquad (n = 1, 2, 3, \ldots)$

(a) ...

(b) ...

(c) ...

You may find the following formula useful:
Energy shift due to perturbation $\epsilon = v_{fi} = \int \psi_f^* \, \hat{v} \, \psi_i \, dx$