THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

June 2015

PHYS3080 Solid State Physics

PHYS3021 Statistical and Solid State Physics

1. Time Allowed: 2 hour
2. Total number of questions: 5
3. Marks available for each question are shown in the examination paper. The total number of marks is 60.
4. Attempt ALL questions!
5. Candidates must supply their own, university approved, calculators.
6. Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
7. The exam paper may be retained by the candidate.

Data and Formula Sheet

NB: This is a generic PHYS3080 data/formula sheet and contains some additional information you may not necessarily need for this exam.

$$e^x = 1 + x + \frac{x^2}{2}\cdots, \qquad \int_0^{\Theta_D/T} \frac{x^4 e^x}{(e^x-1)^2}dx \approx \int_0^{\infty} \frac{x^4 e^x}{(e^x-1)^2}dx \approx \frac{4\pi^4}{15}$$

$$\dot{Q} = \frac{dQ}{dT} = \kappa A \frac{dT}{dx}, \qquad c_V = \tfrac{1}{2}k_B T \text{mole}^{-1} \text{ per degree of freedom}$$

$$\kappa = \tfrac{1}{3}\nu l c, \qquad R = k_B/N_A, \qquad E_{Th.} = k_B T$$

$$\epsilon_{CB}(k) = E_{\text{Gap}} + \frac{\hbar^2 k^2}{2m_e^*}, \qquad \epsilon_{VB}(k) = -\frac{\hbar^2 k^2}{2m_h^*}$$

$$v_{\text{Gr}} = \tfrac{1}{\hbar}\text{grad}_k \epsilon(k), \qquad \tfrac{1}{m^*} = \tfrac{1}{\hbar^2}\tfrac{\partial^2 \epsilon(k)}{\partial k_x^2}$$

$$E_n = -\frac{m_e^* e^4}{8h^2 n^2 \varepsilon_0^2}, \qquad a = a_0 \epsilon_r \left(\frac{m_e}{m_e^*}\right), \qquad a_0 = 0.53\text{Å}$$

$$n_n p_n = n_i^2 = n_p p_p, \qquad n_i = p_i = \sqrt{N_{CB}\cdot N_{VB}}\; exp(-E_{\text{Gap}}/2k_B T),$$

$$n_D \approx N_{CB}\; exp(-E_D/k_B T) \text{ for } k_B T \ll E_D,$$

$$p_A \approx N_{VB}\; exp(-E_A/k_B T) \text{ for } k_B T \ll E_A$$

$$\vec{F} = q(\vec{v}\times\vec{B}), \qquad I = NAve, \qquad v = -\frac{e\tau}{m_e}E$$

$$J = \sigma E, \qquad \sigma = ne\mu = \frac{ne^2\tau}{m_e}, \qquad \mu = \frac{v_d}{E},$$

$$n_{\text{phonon}} \sim \exp(-\Theta_D/T), \qquad \lambda_{\text{phonon}} \sim \exp(+\Theta_D/T)$$

$$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}, \qquad \xi_0 = \frac{\hbar v_F}{\pi\Delta_0}$$

$$\lambda = h/p \quad \text{de Broglie wavelength}$$

$$e = 1.6\cdot 10^{-19}\ C, \qquad \varepsilon_0 = 8.854\cdot 10^{-12}\ Fm^{-1}, \qquad N_A = 6.023\cdot 10^{23}\ g/\text{mole}$$

$$h = 6.63\cdot 10^{-34}\ Js, \qquad \hbar = 1.05\cdot 10^{-34}\ Js, \qquad \lambda_{\text{visible}} \sim 400-700\ nm$$

$$k_B = 1.380658\cdot 10^{-23} J/K,$$

$$m_e = 511\ keV = 9.109\cdot 10^{-31} kg \qquad u = 1.66\cdot 10^{-27} kg$$

Question 1 (10 marks)

Electron Diffraction

(a) In order to determine the crystal structure of a solid, electrons with a wavelength of 1.6 Å can be used. Which voltage would be required to accelerate electrons in order to have a wavelength of 1.6 Å.

(b) Give a brief description of the powder diffraction experiment including a schematic figure. How can you construct a monochromator for electrons?

(c) The distance between the atoms in a cubic crystal is d = 3.2 Å. Calculate the diffraction angles of the first, second, and third order Bragg reflection using the wavelength of 1.6 Å. A flat plate electron detector is placed 30 cm from the crystal. At which distances from each other do the three Bragg refections appear on the detector?

(d) How does the distance between the Bragg peaks change if you double the voltage used to accelerate the electrons.

Question 2 (10 marks)

Explain the following with brief and concise answers!

(a) Give the expression for the reciprocal lattice vectors in terms of the real space vector $\vec{a} = (a_1, a_2, a_3)$. Furthermore, derive the relation between the vectors in real space and reciprocal space by multiplying them, i.e. by multiplying a_i and b_j.

(b) Explain the terms metal, insulator, and semiconductor in terms of band-filling. Give a sketch of each of the different band structures.

(c) Explain why electrons in a completely filled band, in particular in the case of an insulator, do not contribute to the electronic conductivity.

(d) Sketch the temperature dependence of the metallic conductivity and explain what phenomena determine its properties. In particular explain why the conductivity increases linearly with temperature ($\propto T$) at high temperatures and explain the origin of the residual resistance at lowest temperatures.

(e) Sketch the setup for a photoluminescence experiment and explain the electronic processes involved using a diagram of the valence band maximum and conduction band minimum.

Question 3 (20 marks)

(a) Density of States of a free Electron

Calculate the density of states ($D(\epsilon)$) of free electrons using the known energy dispersion ($\epsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m_e}$) for the 1-, 2-, and 3-dimensional case.

(b) Fermi Energy

(i) Derive the expression of the Fermi vector and Fermi energy of a free three dimensional electron gas:

$$k_F = \sqrt[3]{3\pi^2 \frac{N}{V}} \qquad E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

(ii) Calculate the Fermi vector k_F and Fermi energy E_F for metallic aluminium (Al is trivalent with a density of $\varrho_{Al} = 2.70 \cdot 10^3 kg/m^3$ and an atomic mass of 27.0 $kg/kmole$). Give your answer in electron volts.

(iii) Calculate the Fermi temperature, Fermi velocity, and the de-Broglie wavelength of an electron moving in aluminium at the Fermi energy.

Question 4 (10 marks)

Specific Heat of a Insulator

(a) Give the equation of the Bose-Einstein distribution function and explain all symbols in this expression.

(b) What is the Debye model and what is the Einstein model? Explain both models and sketch the Density of States as a function of energy. Which phonons are represented by the Debye model and which by the Einstein model?

(c) Calculate the specific heat in the case of the Einstein model with a density of states of $D(\omega) = 3N \cdot \delta(\omega - \omega_E)$:

$$U(T) = \sum_{\nu} D(\omega)\, \hbar\omega\, (f_{\text{BE}}(E_\nu, T) + 1/2)$$

$$c_V(T) = \left(\frac{d\, U_{\text{osz.}}(T)}{dT}\right)_T$$

(d) Plot the specific heat of an insulator in the entire temperature range down to T = 0 K.

(e) In a few words: How does the temperature dependence of the specific heat change in the case of a metal such as copper?

Question 5 (10 marks)

Thermal Conductivity

(a) Sketch and explain a setup to measure the thermal conductivity.

(b) The following figure (Fig. 1) shows the thermal conductivity of silicon. Explain which processes dominate the properties in the:

(i) low temperature region below the peak maximum,

(ii) the high temperature region above the maximum,

(c) How can the height of the peak maximum be increased, as shown in the experimental data given in Figure 1.

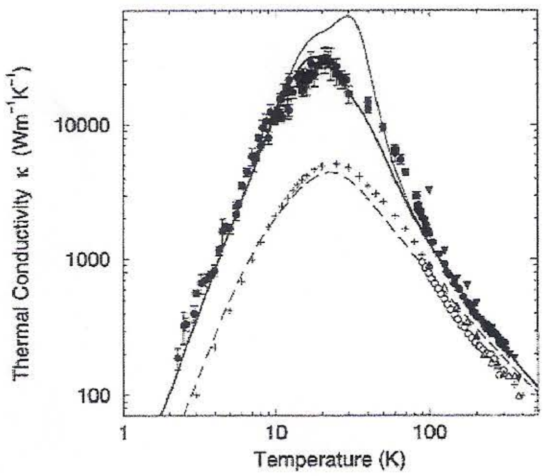

Figure 1: Temperature dependence of the thermal conductivity of Silicon. Note that the data are given on a double logarithmic scale.