THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

June/July 2013

PHYS3080 Solid State Physics

PHYS3021 Statistical and Solid State Physics

1. Time Allowed: 2 hour
2. Total number of questions: 5
3. Marks available for each question are shown in the examination paper. The total number of marks is 60.
4. Attempt ALL questions!
5. University-approved calculators may be used.
6. Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
7. The exam paper may be retained by the candidate.

Data and Formula Sheet

N.B. This is a generic PHYS3080/PHYS3021 data/formula sheet

$\mathbf{a}^* = \frac{2\pi(\mathbf{b}x\mathbf{c})}{\mathbf{a}.(\mathbf{b}x\mathbf{c})}$ and cyclic permutation of numerator

$e^x = 1 + x + \frac{x^2}{2}\ldots \qquad \int_0^{\Theta_D/T}\left(\frac{x^4e^xdx}{(e^x-1)^2}\right) \cong \int_0^{\infty}\left(\frac{x^4e^xdx}{(e^x-1)^2}\right) = \frac{4\pi^4}{15}$

$\dot{Q} = \frac{dQ}{dt} = \kappa A\frac{dT}{dx} \qquad C_v = 1/2\,k_BT\ \text{mol}^{-1}$ per degree of freedom

$\kappa = \frac{1}{3}\bar{v}lC \qquad R = k_B/N_A \qquad E_{th} = k_BT$

$\varepsilon = E_g + \frac{\hbar^2k^2}{2m_e} \qquad \varepsilon = -\frac{\hbar^2k^2}{2m_h} \qquad E_n = -\frac{m_e^*e^4}{8h^2n^2\varepsilon_0^{\,2}} \qquad a = a_0\varepsilon_r\left(\frac{m_e}{m_e^*}\right) \qquad a_0 = 0.053\text{ nm}$

$n_np_n = n_i^{\,2} = n_pp_p \qquad R_H = -\frac{1}{ne} \qquad n_i = p_i = (N_cN_v)^{1/2}\exp(-E_g/2k_BT)$

$np = (N_cN_v)\exp(-E_g/k_BT)$

$n \approx N_c\exp(-E_D/k_BT)$ for $k_BT << E_D \qquad p \approx N_v\exp(-E_A/k_BT)$ for $k_BT << E_A$

$\mathbf{F} = q(\mathbf{v}x\mathbf{B}) \qquad I = nAve \qquad \mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = ne\mu = \frac{ne^2\tau}{m}$

$\mu = \frac{v_d}{E}$

$e = 1.6x10^{-19}\text{ C} \qquad \varepsilon_0 = 8.854x10^{-12}\text{ Fm}^{-1} \qquad N_A = 6.023x10^{26}\ (\text{kg.mol})^{-1}$

$h = 6.63x10^{-34}\text{ Js} \qquad \hbar = 1.05x10^{-34}\text{ Js} \qquad \hbar^2 = 1.11x10^{-68}\text{ J}^2\text{s}^2 \qquad \lambda_{visible} \sim 400-700\text{nm}$

$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk_x} \qquad m^* = \hbar^2\Big/\frac{d^2\varepsilon}{dk_x^2} \qquad j = j_0\sin\left[\frac{2e}{\hbar}\left(V_0t + \frac{V}{\omega}\sin(\omega t)\right) + \delta_0\right]$

$V_0 = \frac{n\hbar\omega}{2e} = \frac{nh\nu}{2e}$

$n_{phonon} \sim \exp(-\Theta_D/T) \qquad \lambda_{phonon} \sim \exp(+\Theta_D/T)$

$k_F = \left(\frac{3\pi^2N}{V}\right)^{1/3} \qquad \xi_0 = \frac{\hbar v_F}{\pi\Delta(0)} \qquad V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi$

Question 1 (10 marks)

Energy Dispersive Neutron Diffraction

In an energy-dispersive neutron scattering experiment the scattering vector is fixed and the velocity, i.e. energy, of the scattered neutron is measured. This is performed in a so-called time-of-flight experiment.

What is the velocity and hence the energy (in eV) of neutrons which are scattered at the (2,0,0) and (10,0,0) Bragg peaks of a rocksalt crystal NaCl (simple cubic crystal structure with $a = 5.63$ Å) if the Bragg scattering angle is fixed at $\theta = 90°$? Mass of the neutron: $1.675 \cdot 10^{-27}$ kg.
Hint: calculate first the wavelength using Braggs law ($2d \sin \Theta = n\lambda$).

Is it possible to perform energy-dispersive time-of-flight X-ray experiments?

Question 2 (10 marks)

Explain the following, each in a few words

(a) Give the expression for the reciprocal lattice vectors and explain the relation between the vectors in real space and reciprocal space by multiplying them.

(b) How can phonons be determined experimentally? Give ONE example and explain this technique briefly, including a sketch of the experimental setup.

(c) How does the dispersion of a free electron change in the case of a conduction electron in a periodic lattice. Sketch the electron dispersion relation in the first Brillouin zone for both cases.

(d) Explain the terms metal, insulator, semiconductor, and semi-metal in terms of band-filling. Give a sketch of each of the different band structures.

(e) What is a direct and what is an indirect band-gap semiconductor?

Question 3 (12 marks)

(a) Density of States of a free Electron

Calculate the density of states ($D(\epsilon)$) of free electrons using the known energy dispersion ($\epsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m_e}$) for the 1-, 2-, and 3-dimensional case.

(b) Dispersion Relation

The following dispersion relation is given:

$$\epsilon(\vec{k}) = \frac{\hbar^2}{2m}\left[\left(\frac{k_x}{1}\right)^2 + \left(\frac{k_y}{2}\right)^2 + \left(\frac{k_z}{3}\right)^2\right]$$

Calculate the group velocity in the plane $k_z = 0$ and plot the dispersion relation in this plane.

Question 4 (15 marks)

Specific Heat of a Insulator

(a) Give the equation of the Bose-Einstein distribution function and explain all symbols in this expression.

(b) What is the Debye model and what is the Einstein model. Explain both models and sketch the Density of States as a function of energy. Which phonons are represented by the Debye model and which by the Einstein model?

(c) Calculate the specific heat in the case of the Einstein model.

$$U_{\text{osz.}}(T) = \sum_{\nu} e_\nu(f_{\text{BE}}(E_\nu, T) + 1/2)$$

$$c_V(T) = \left(\frac{d\,U_{\text{osz.}}(T)}{dT}\right)_T$$

(d) Plot the specific heat of an insulator down to T = 0 K.

(e) In a few words: How does the specific heat change in the case of a metal such as copper?

Question 5 (13 marks)

Fermi Surface

Silver crystallizes in the face centered cubic crystal structure and possesses one conduction electron per atom. Its density is $\varrho = 10.6\ g/cm^3$ and the mass of one atom is $m_{Ag} = 107.9\ u$ where $u = 1.66054 \cdot 10^{-27} kg$.

Calculate:

(a) the radius of the Fermi sphere,

(b) the Fermi energy and Fermi temperature,

(c) the size of the unit cell (lattice parameter),

(d) the length of the reciprocal lattice vector,

(e) and the volume of the first Brillouin zone.

Note that the Fermi energy of a free electron gas at $T = 0\ K$ is given by:

$$\epsilon_{\mathrm{F}}(T = 0K) = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$$