THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2011

PHYS3080/9783

Solid State Physics

Time Allowed – 2 hours

Total number of questions - 5

Answer ALL questions

All questions are NOT of equal value

This paper may be retained by the candidate.

Students must provide their own UNSW approved calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work

Data and Formula Sheet

N.B. This is a generic PHYS3080/9783 data/formula sheet and contains some additional information you may not necessarily need for this exam

$$\mathbf{a}^* = \frac{2\pi(\mathbf{b}\mathrm{x}\mathbf{c})}{\mathbf{a}.(\mathbf{b}\mathrm{x}\mathbf{c})}$$ and cyclic permutation of numerator

$$e^x = 1 + x + \frac{x^2}{2}\ldots \qquad \int_0^{\Theta_D/T}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) \cong \int_0^{\infty}\left(\frac{x^4 e^x dx}{(e^x-1)^2}\right) = \frac{4\pi^4}{15}$$

$$\dot{Q} = \frac{dQ}{dt} = \kappa A \frac{dT}{dx} \qquad C_v = 1/2\, k_B T \text{ mol}^{-1} \text{ per degree of freedom}$$

$$\kappa = \frac{1}{3}\bar{v} l C \qquad R = k_B/N_A \qquad E_{th} = k_B T$$

$$\varepsilon = E_g + \frac{\hbar^2 k^2}{2m_e} \qquad \varepsilon = -\frac{\hbar^2 k^2}{2m_h} \qquad E_n = -\frac{m_e^* e^4}{8h^2 n^2 \varepsilon_0{}^2} \qquad a = a_0 \varepsilon_r \left(\frac{m_e}{m_e^*}\right) \qquad a_0 = 0.053\text{ nm}$$

$$n_n p_n = n_i{}^2 = n_p p_p \qquad R_H = -\frac{1}{ne} \qquad n_i = p_i = (N_c N_v)^{1/2} \exp(-E_g/2k_B T)$$

$$np = (N_c N_v)\exp(-E_g/k_B T)$$

$$n \approx N_c \exp(-E_D/k_B T) \text{ for } k_B T << E_D \qquad p \approx N_v \exp(-E_A/k_B T) \text{ for } k_B T << E_A$$

$$\mathbf{F} = q(\mathbf{v}\mathrm{x}\mathbf{B}) \qquad I = nAve \qquad \mathbf{v} = -\frac{e\tau}{m_e}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E} \qquad \sigma = ne\mu = \frac{ne^2\tau}{m}$$

$$\mu = \frac{v_d}{E}$$

$$e = 1.6\text{x}10^{-19}\text{ C} \qquad \varepsilon_0 = 8.854\text{x}10^{-12}\text{ Fm}^{-1} \qquad N_A = 6.023\text{x}10^{26}\ (\text{kg.mol})^{-1}$$

$$h = 6.63\text{x}10^{-34}\text{ Js} \qquad \hbar = 1.05\text{x}10^{-34}\text{ Js} \qquad \hbar^2 = 1.11\text{x}10^{-68}\text{ J}^2\text{s}^2 \qquad \lambda_{visible} \sim 400 - 700\text{nm}$$

$$v = \frac{1}{\hbar}\frac{d\varepsilon}{dk_x} \qquad m^* = \hbar^2 \Big/ \frac{d^2\varepsilon}{dk_x^2} \qquad j = j_0 \sin\left[\frac{2e}{\hbar}\left(V_0 t + \frac{V}{\omega}\sin(\omega t)\right) + \delta_0\right]$$

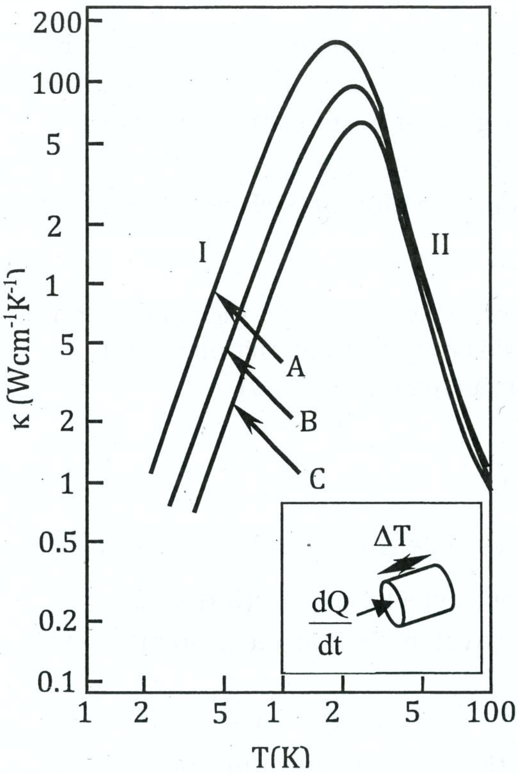
$$V_0 = \frac{n\hbar\omega}{2e} = \frac{nh\nu}{2e}$$

$$n_{phonon} \sim \exp(-\Theta_D/T) \qquad \lambda_{phonon} \sim \exp(+\Theta_D/T)$$

$$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3} \qquad \xi_0 = \frac{\hbar v_F}{\pi\Delta(0)} \qquad V_0 = \frac{n\hbar\omega}{2e} = n\nu\Phi$$

Question 1 (18 Marks)

The diagram below shows thermal conductivity data as a function of temperature for three pure, crystalline specimens of LiF labeled A, B and C. The inset (bottom right) indicates schematically how the measurements were performed: a constant heat current $\dot{Q}$ into the crystal face of area A produced a measured temperature difference ΔT along the specimen. In region I the thermal conductivity follows a T^3 law.



(i) Assuming A, B and C to have identical chemical composition, morphology and purity, what is the probable cause of the difference seen in the data for the three specimens below T~20K; explain briefly (no more than 4 or 5 lines). **(6 marks)**

(ii) The kinetic theory (for classical gases) expression for thermal conductivity κ must be modified when applied to solids and for different temperature ranges. What are the physical parameters and/or mechanisms determining κ in regions I and II (marked on graph) respectively? Provide a brief (no more than 4-5 lines) explanation for each. **(6 marks)**

(iii) For a LiF specimen in the form of a right circular cylinder, diameter 4 mm x length 1cm, *estimate* the heat current required to produce a temperature difference of 0.05 K along the length of the crystal at T~50K. **(6 marks)**

Question 2 (20 Marks)

(a) The Fermi-Dirac distribution function

$$f(\varepsilon) = \frac{1}{1+\exp\left(\frac{\varepsilon-\varepsilon_F}{k_B T}\right)}$$

gives the state occupation probability for electrons in a free electron metal.
(i) Define all symbols in this expression. **(3 marks)**
(ii) The total number of occupied electron states, $N(\varepsilon)$, in the energy range $\varepsilon \rightarrow \varepsilon + d\varepsilon$ is given by the product of the occupation probability $f(\varepsilon)$ with the density states function $g(\varepsilon)$. Sketch the form of the three quantities $N(\varepsilon)$, $f(\varepsilon)$, $g(\varepsilon)$ for a simple free electron metal. Indicate the situation for $T = 0K$ and $T_F >> T >> 0K$, where T_F is the Fermi temperature. (Put both curves on one plot or use a separate plot for each, as you prefer.) **(6 marks)**

(b) Give a concise explanation of the reason the observed electronic (i.e. the conduction electrons) contribution to the heat capacity of a metal is only a small fraction of that expected classically. Include a sketch illustrating your answer. **(4 marks)**

(c) The Fermi energy at 0K is given by $\varepsilon_{F,0} = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$.
(i) Calculate $\varepsilon_{F,0}$ for aluminium metal (Al is *trivalent* with density $\rho_{Al} = 2.70\text{x}10^3\ \text{kgm}^{-3}$ and atomic mass $26.98\ \text{kg}(\text{kmole})^{-1}$); give your answer in electron volts. **(3 marks)**
(ii) Determine the Fermi velocity and the de Broglie wavelength of an electron moving in aluminium at the Fermi energy. **(2 marks)**
(d) A particular sample of aluminium has drift velocity $v_d = 2.16\ \text{ms}^{-1}$ in an electric field $E = 500\ \text{Vm}^{-1}$. Estimate (i) the electron mobility; (ii) the relaxation (scattering) time. **(2 marks)**

Question 3 (22 Marks)

Consider a crystal for which the $\varepsilon - k$ relation is $\varepsilon = \varepsilon_1 + (\varepsilon_2 - \varepsilon_1)\sin^2(ak_x/2)$, with $\varepsilon_2, \varepsilon_1$ constants.

(a) Sketch and label the $\varepsilon - k$ dispersion relation for the first two Brillouin zones. **(4 marks)**

(b) Populate the band with one electron (i.e. assume one electron in the band), and, ignoring scattering of the electron and sketch and describe the behaviour of

(i) the effective mass, **(6 marks)**

(ii) the electron velocity, **(6 marks)**

in a constant applied electric field E. Sketch each on a separate graph. Your sketch should include the range $-\pi/a < k_x \leq \pi/a$.

(c) Briefly describe the motion of the electron in this band when a constant electric field E is applied. **(6 marks)**

Question 4 (20 Marks)

(a) The electrical conductivity σ of a doped semiconductor specimen is measured over a wide temperature range with the following features being observed:

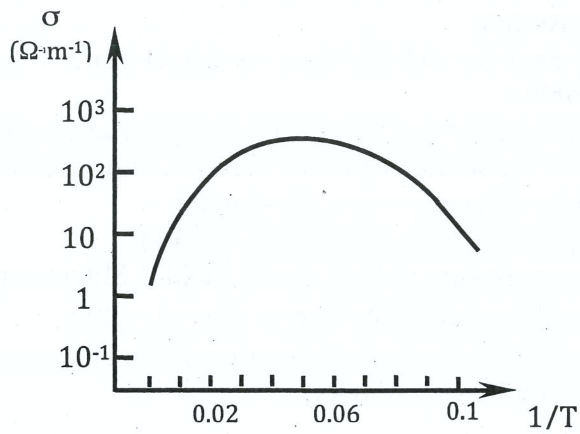
(i) At low temperatures σ rises with increasing temperature.

(ii) At intermediate temperatures σ falls with increasing temperature.

(iii) At high temperatures σ increases rapidly with increasing temperature.

The behaviour is illustrated on the graph at right; region (iii) is not shown.

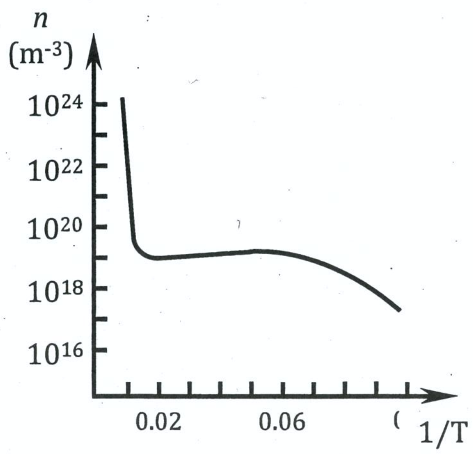
Give the probable reasons for the behaviour in each temperature region. **(10 marks)**



(b) A Hall effect measurement is performed over a wide temperature range on the specimen discussed in part (a) above. The carrier concentration *n* in different temperature regions is determined from this measurement.

(i) From which temperature region would you estimate the band gap E_g? Give the reasons for your choice and state what you would plot graphically to find a value for E_g. **(5 marks)**

(ii) From which temperature region would you estimate the net donor concentration N_D-N_A? Give the reasons for your choice and estimate the value of N_D-N_A for this specimen. **(5 marks)**



Question 5 (20 Marks)

Write brief notes (about 2-3 pages for each, including diagrams and any equations you include, but no more than this) on ***two only*** from the list of four topics given below.

Use simple diagrams and/or sketch graphs to illustrate your answers, where appropriate, ensuring that you label these and refer to them in your account.

Choose ***two only*** from the following four (a) – (d)

(a) Crystalline solids. Your answer must discuss (but is not limited to) lattice and basis, bravais lattices and symmetry, conventional and primitive unit cells, packing fraction. **(10 marks)**
(b) Heat capacity of solids. Your answer must discuss (but is not limited to) 'classical' theory (law of Dulong and Petit), the Debye and Einstein phonon models and the electronic (electron) contribution to the heat capacity. **(10 marks)**
(c) Effect of dopant concentration and temperature on the electrical conductivity of semiconductor materials. The relevant sketch graphs must be included in your answer. **(10 marks)**
(d) The Josephson effects. You may use information from the data sheet and refer to the diagram included below. **(10 marks)**