THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – SEPTEMBER 2015

PHYS3031 – ADVANCED OPTICS AND NUCLEAR PHYSICS Paper 2

PHYS3060 – ADVANCED OPTICS

Time allowed – 50 minutes

Total number of questions – 2

Attempt **ALL** questions

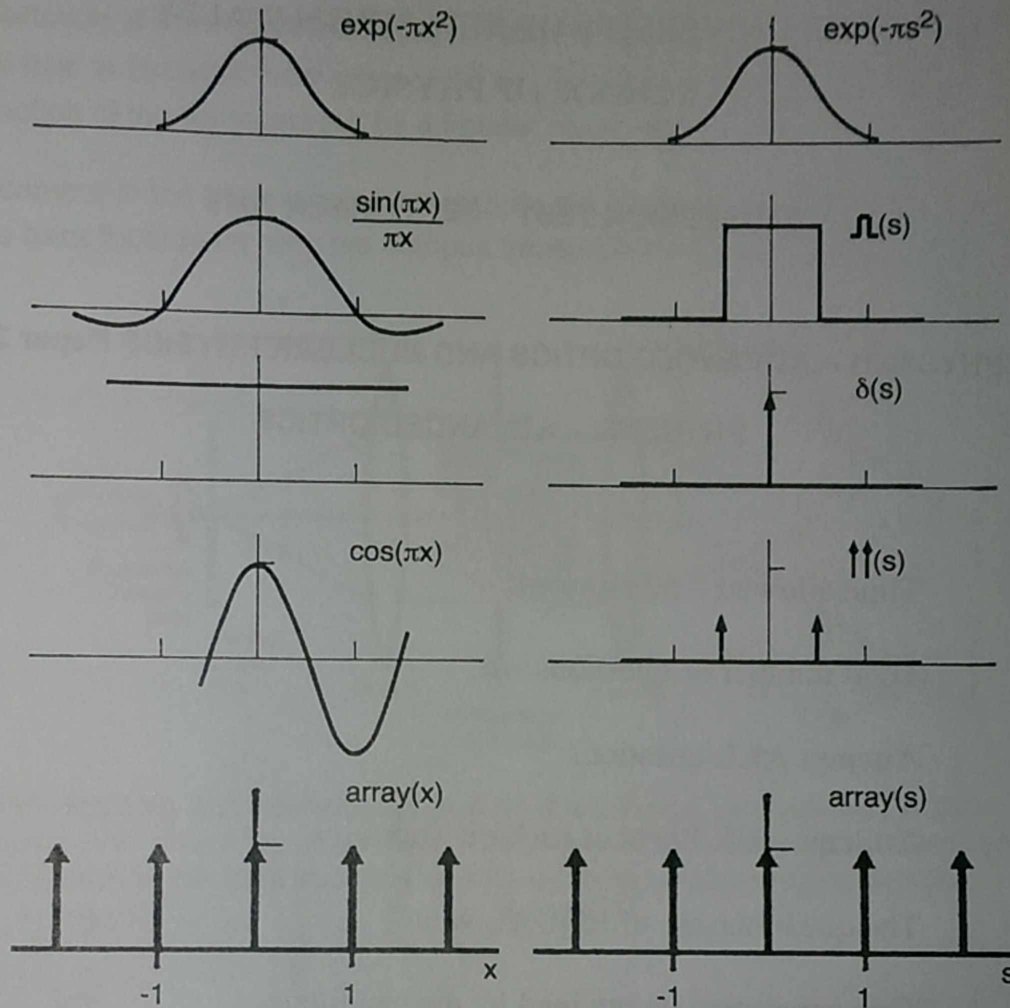
Attempt **ALL Parts** of each question

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.



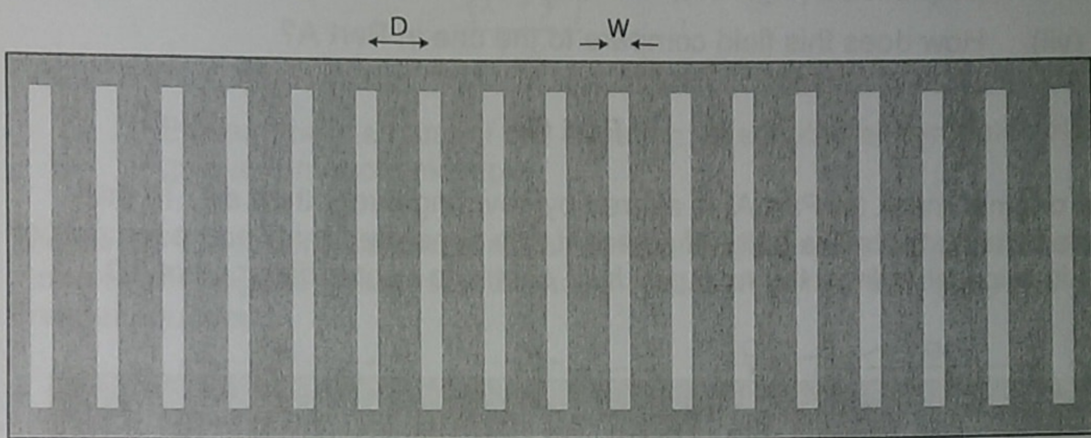
Theorems of the Fourier Transform

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a \rvert} F\left(\frac{s}{a}\right)$
Linearity	$\lambda f(x) + \mu g(x)$	$\lambda F(s) + \mu G(s)$
Shift	$f(x-a)$	$e^{-2\pi i a s} F(s)$
Convolution	$f(x) \otimes g(x)$	$F(s) \bullet G(s)$

Question 1

PART A

A series of very long slits are cut into an opaque mask (i.e. treat this as a one-dimensional problem and assume an infinite number of slits). The width of each slit is W and they are separated by a centre-to-centre distance of D.



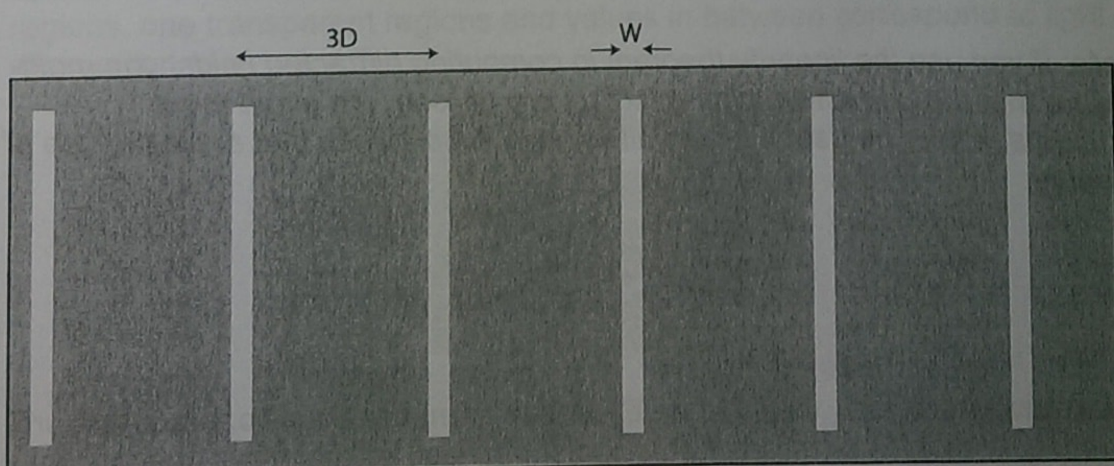
(i) Write a one-dimensional transmission function to describe the properties of this mask.

(ii) Using Fourier theory (or otherwise), derive an expression for the Fraunhofer diffracted **field** produced by this mask.

(iii) Sketch the Fraunhofer diffracted **field** as a function of s, the Fourier variable. Label the positions of each zero and each δ function on your sketch in terms of spatial frequencies (e.g. 1/W, -2/D etc).

PART B

A new mask is made which still consists of an essentially infinite number of long slits (width W), however, this time, the separation between the slits is increased to a centre-to-centre distance of 3D.



(iv) Write a one-dimensional transmission function for this new mask.

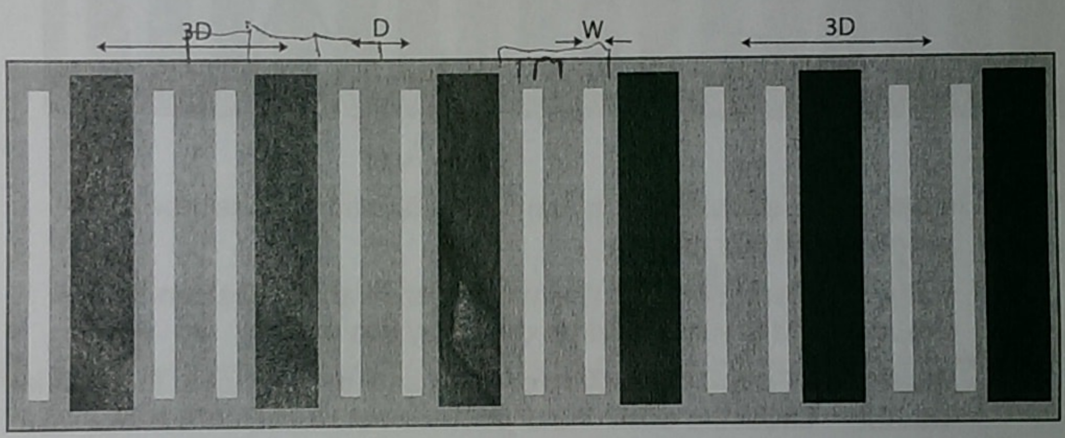
(v) Derive an expression for the Fraunhofer diffracted **field** produced by this new mask.

(vi) Sketch the Fraunhofer diffracted **field** produced by this new mask as a function of the Fourier variable s. Label the positions of each zero and each δ function on your sketch in terms of spatial frequencies (e.g. 1/W, -2/D etc).

(vii) How does this field compare to the one in Part A?

Part C

The original mask (in Part A) is altered by covering every third slit. In this altered mask, there are pairs of adjacent slits separated by D and each pair of slits is separated from the next pair by a centre-to-centre distance of 3D.



(viii) Derive an expression for the diffracted **field** produced by this new arrangement.

(ix) Sketch the Fraunhofer diffracted **field** produced by this new arrangement.

(x) Comment on how the **intensity** pattern would compare with that produced by the original, simple mask in Part A.

Note: *If you use the linearity theorem in computing diffracted fields, you must ensure that the correct weighting factors are utilised. To avoid these problems, it may be useful to find alternative descriptions that avoid the use of linearity.*

QUESTION 2

Part A

The Fraunhofer or far field diffraction pattern is related to the transmission function of an object by its Fourier transform. The Fraunhofer condition is given by:

$$\frac{(x^2 + y^2)}{2\lambda R_o} \ll 1$$

(i) Explain the meaning of this equation, describing all symbols AND give a numerical example.

The above Fraunhofer condition arises as the result of an approximation made in deriving the Fraunhofer diffraction equation from the Huygens-Fresnel equation.

(ii) Explain why this condition is required to make the appropriate approximations that result in the Fraunhofer diffraction equation.

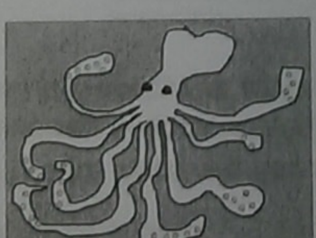
Several systems of variables are used to describe Fraunhofer diffraction. These include the distances (ζ,η), the angles (θ,ϕ) and the Fourier space variables (u, v).

(iii) Define the relationships between each of these systems of variables. Use a sketch to show these relationships.

(iv) Explain their physical significance and indicate why they are each useful.

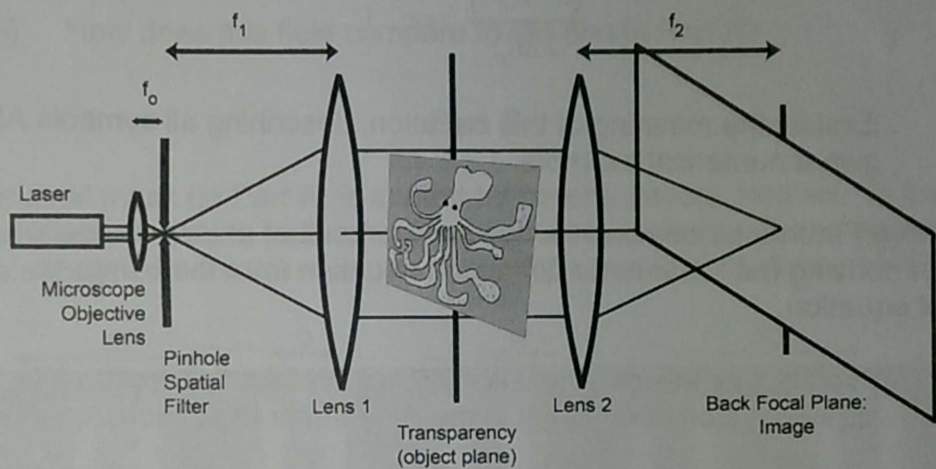
Part B

A transparency is created, showing a cartoon of an octopus. The transmission function describing the octopus transparency, $f(x,y)$, is a real function bounded between zero and one, where zero corresponds to opaque regions, one transparent regions and values in between correspond to light attenuation.

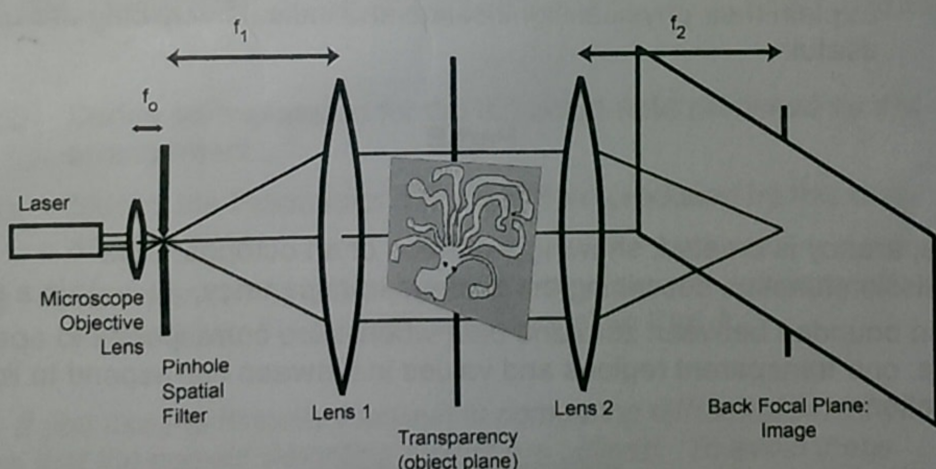


The transparency is placed in an optical diffractometer. This is a device that allows one to examine the light intensity distribution in the back focal plane as a function of the coordinates (ζ,η). From Abbe's theory of image formation, the field in the back focal plane, $\psi(\zeta,\eta)$, is related to the transmission function of the transparency by a Fourier transform.

A camera in the back focal plane records the intensity distribution, $I(\zeta,\eta)$, in the back focal plane with the octopus transparency in front of the lens.



After recording this intensity distribution, the octopus transparency is then rotated 180° about the optical axis. The camera then records the intensity distribution in the back focal plane for the rotated octopus transparency, $I_{180^\circ}(\zeta,\eta)$.



(v) Show that the intensity distribution in the back focal plane is identical for the original octopus transparency and the 180° rotated octopus transparency, i.e. show that $I(\zeta,\eta)=I_{180^\circ}(\zeta,\eta)$

(vi) How could you alter the octopus transparency so as to break this symmetry?