THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION EXAMINATION

SEPTEMBER 2014

PHYS3031/PHYS3060

ADVANCED OPTICS

Time Allowed – 50 minutes

Total number of questions – 3

Answer ALL questions

Questions are **NOT** of equal value

Candidates should provide their own university approved calculator

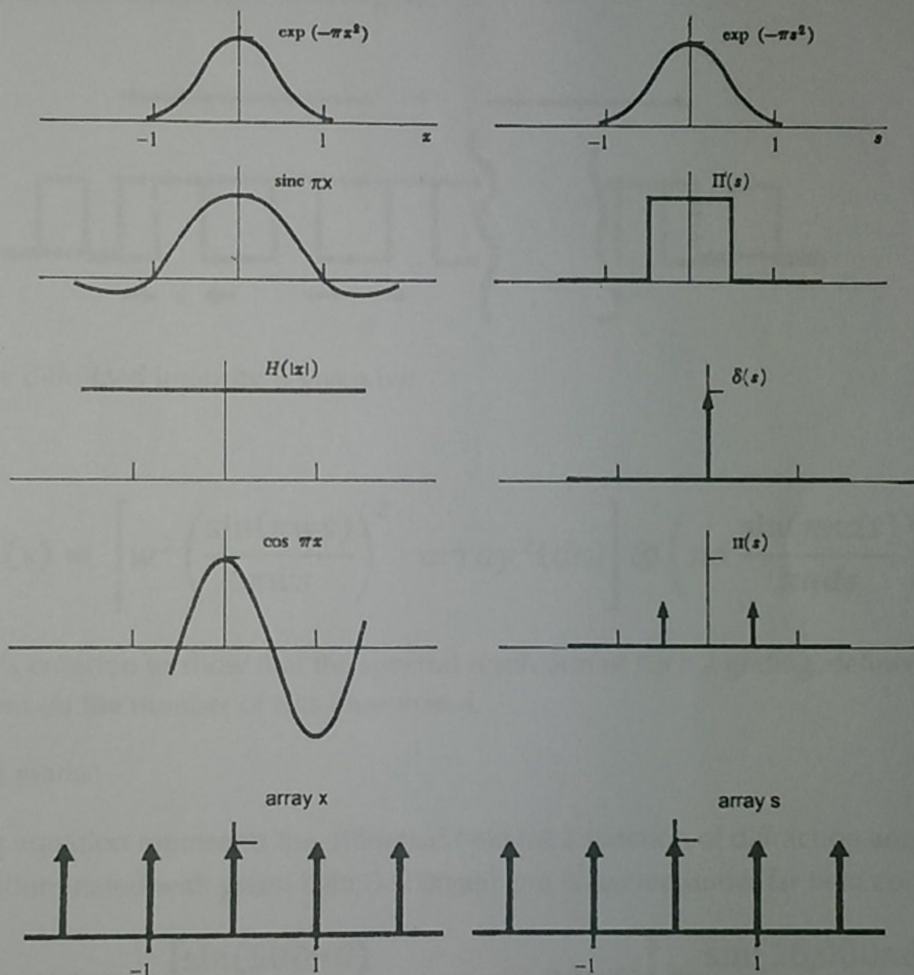
Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Candidates may keep this examination masterpiece.

$$F(s) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ixs}\,dx, \quad f(x) = \int_{-\infty}^{\infty} F(s)e^{2\pi ixs}\,ds$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$F\{f(x)\} = 2\int_{0}^{\infty} E(x)\cos(2\pi sx)\,dx - 2i\int_{0}^{\infty} O(x)\sin(2\pi sx)\,dx$$



$\cos\pi x\, e^{2\pi ixs}\, ds$

$= \cos\pi x\, e^{-2\pi ixs}\, dx$

$= \frac{e^{\pi x} - e^{-\pi x}}{2} \cdot e^{-2\pi ixs}\, dx$

$= \frac{e^{x(\pi - 2\pi is)} - e^{-x(\pi - 2\pi is)}}{2}\, dx$

$=$

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a\rvert}F(\frac{s}{a})$
Linearity	$\alpha f(x) + \beta g(x)$	$\alpha F(s) + \beta G(s)$
Shift	$f(x-a)$	$e^{-i2\pi as}F(s)$
Convolution	$f(x)\otimes g(x)$	$F(s)G(s)$

$s = \frac{\sin\theta}{\lambda}$

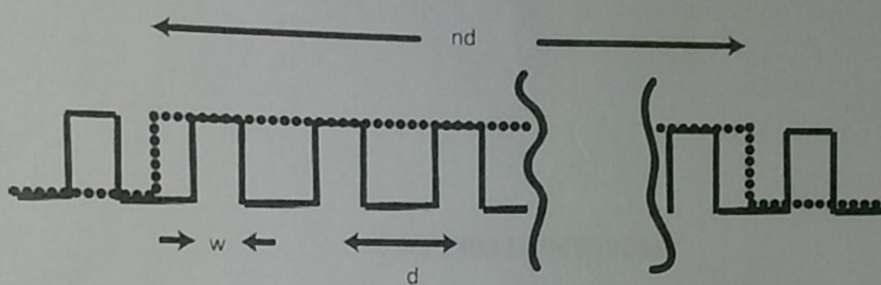
$n\lambda = d\sin\theta$

Question 1 (3 marks)

Derive the Fourier transform of the double delta function.

Question 2. (6 marks)

Consider a diffraction grating consisting of a finite number of evenly spaced slits, where the relevant dimensions are indicated in the figure.



Given that the diffracted intensity is given by:

$$I(s) = \left[w^2\left(\frac{\sin(\pi ws)}{\pi ws}\right)^2 \cdot array^2(ds)\right] \otimes \left(nd\frac{\sin(\pi nds)}{\pi nds}\right)^2$$

Use Rayleigh's criterion to show that the spectral resolution of such a grating, defined as $\Delta\lambda/\lambda$, is only dependent on the number of slits illuminated.

Question 3 (9 marks)

The following equation represents the diffracted field (as a function of diffraction angle) from a finite grating illuminated with green light (λ=500nm) and observed under far field conditions:

$$\phi(\theta) \propto \left[\frac{\sin(500\pi\theta)}{500\pi\theta} \cdot array(2{,}000\theta)\right] \otimes \frac{\sin(16{,}000\pi\theta)}{16{,}000\pi\theta}$$

(i) Determine an expression for the aperture function, including values for the slit width, array spacing and number of periods. Provide your answer in spatial dimensions (i.e. mm).

(ii) An opaque mask is placed over every **second** array position of the diffraction pattern and a thin spherical lens is placed in front of the filtered field. Derive an expression for the field observed at the back focal plane of the lens. Recall that a lens performs a Fourier transform. Assume unity magnification of the image.

(iii) Use the result in (ii) to determine the slit width, spacing and number of periods in the filtered image of the aperture (observed at the back focal plane of the lens). Provide your answers in spatial dimensions.