THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2011

PHYS3060 – ADVANCED OPTICS

Time allowed – 2 hours

Total number of questions – 4

Attempt **ALL** questions

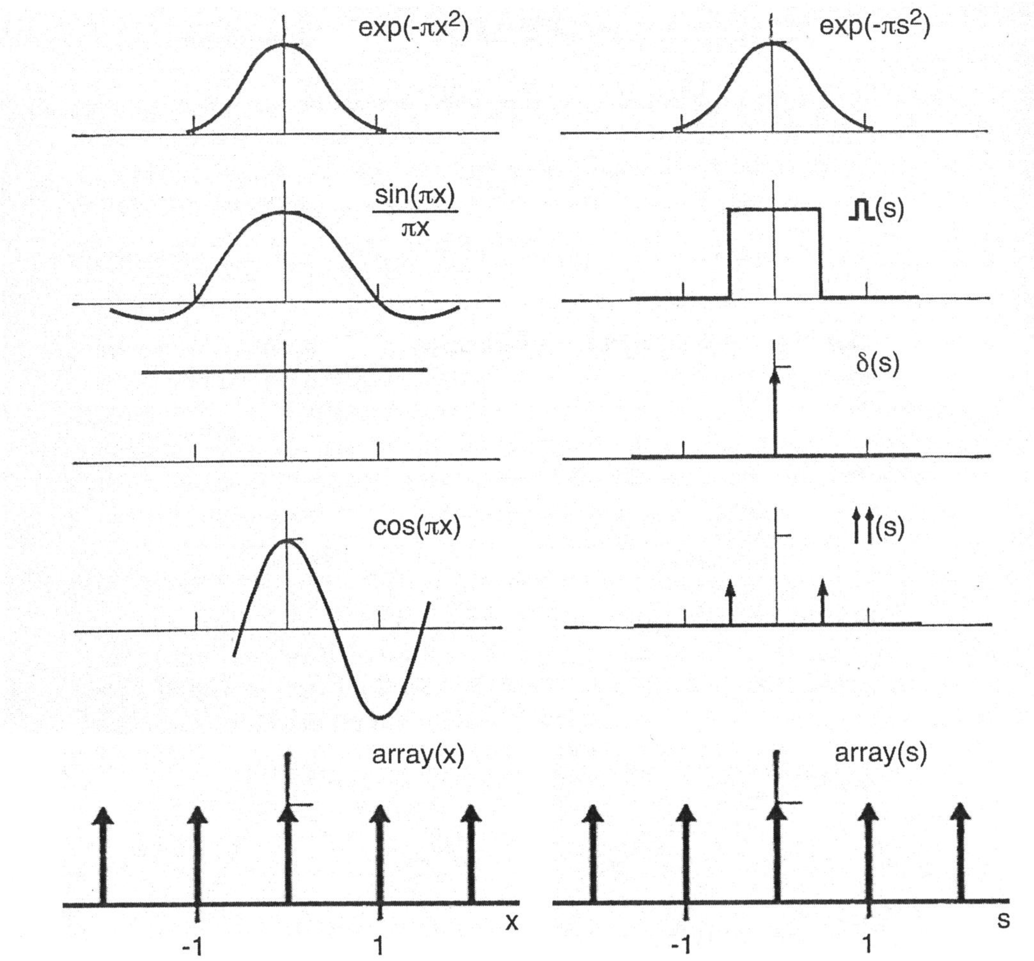
Attempt **ALL Parts** of each question

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.



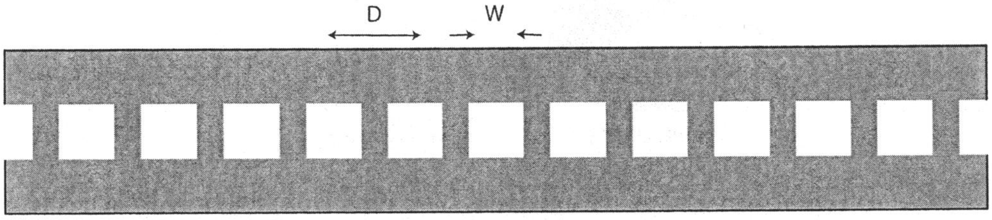
Theorems of the Fourier Transform

Theorem	$f(x)$	$F(s)$
Similarity	$f(ax)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$
Linearity	$\lambda f(x)+\mu g(x)$	$\lambda F(s)+\mu G(s)$
Shift	$f(x-a)$	$e^{-2\pi i a s}F(s)$
Convolution	$f(x)\otimes g(x)$	$F(s)\bullet G(s)$

Question 1

PART A

A mask is made by cutting a large number of square holes (side W) in a line where the centre-to-centre separation between square holes is D.

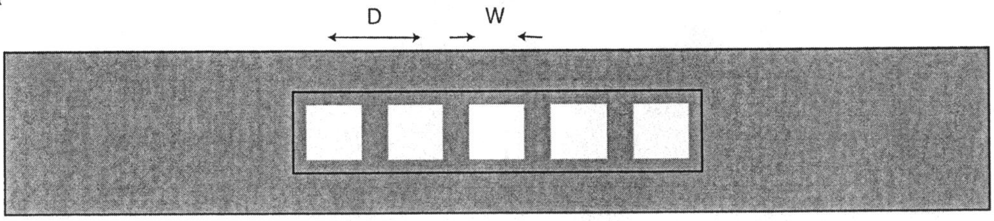


(i) Write an expression for the transmission function of this mask.

(ii) Using Fourier theory (or otherwise) derive an expression for the diffracted field produced by this mask.

(iii) Make two graphs, one showing the diffracted intensity in the horizontal direction (parallel to the line of holes) and the other showing the diffracted intensity in the vertical direction (perpendicular to the line of holes). Mark the positions of zeros and maxima in the diffracted intensity.

The original mask is covered with an opaque sheet with a central rectangular hole that allows light to only pass through the central five square holes in the original array of square holes.



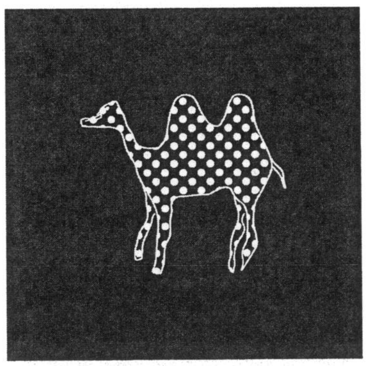
(iv) Derive an expression for the Fraunhofer diffracted field produced by the mask with five square holes.

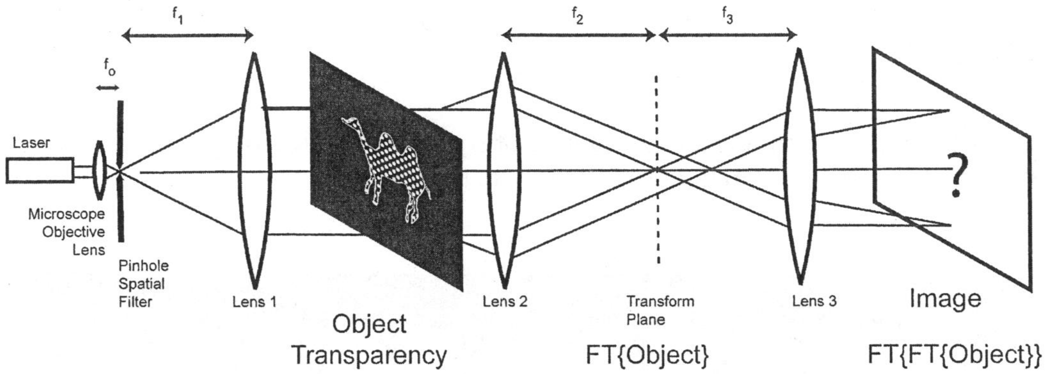
(v) Make a graph of the diffracted intensity in the horizontal direction (parallel to the original array of square holes). Mark the positions of zeros and maxima.

PART B

A diffraction transparency is produced by first producing a mask consisting of a two dimensional array of small holes (assume that these holes can be approximated as Dirac δ functions). This hole array is then covered with the cut out of a camel.

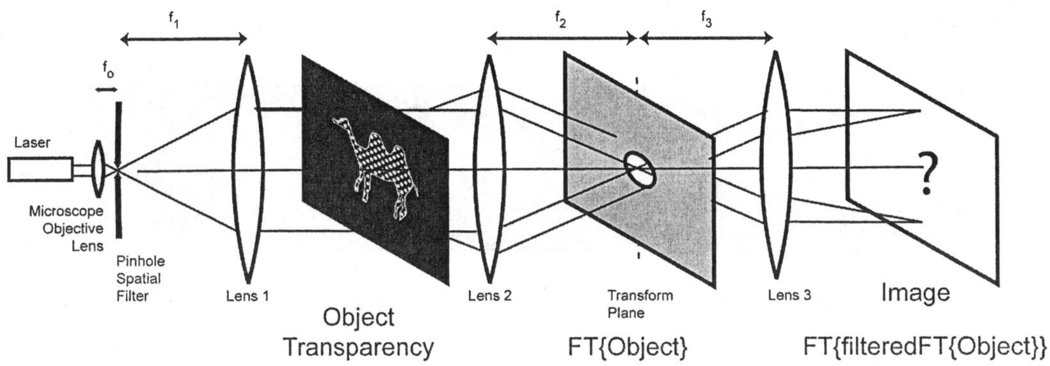
This mask is placed in the object plane on an optical Fourier transformer, where the optical field in the back focal plane of Lens 2 is related to the transmission function of the mask by a Fourier transform.





(vi) Using Fourier theory or otherwise, describe the Fraunhofer diffracted field produced by this transparency (camel shape sampled by a 2D array of δ functions). This is the field in the back focal plane of Lens 2.

A filtering mask is inserted into the diffraction plane (back focal plane of Lens 2). This mask consists of a large circular hole that only allows the strong central maximum to be transmitted plus its surrounding darkish region but not any additional bright features in the diffracted field arising from the Fourier transform of the transparency.



(vii) Using Fourier theory (or otherwise), determine the new image produced by Lens 3 after the diffracted field is filtered by the circular hole mask.

Question 2

Answer **TWO** of the following questions. Use words, pictures and/or equations to illustrate your answer. Give numerical examples wherever possible.

(i) The Fourier transform of an object decomposes it into spatial frequencies. Fraunhofer diffraction theory shows that the diffracted field produced by mask which is small compared to the distance between the mask and the observation plane is given by a Fourier transform. Explain why the diffraction pattern is related to the spatial frequencies in the diffraction mask. Give examples.

(ii) Explain the mathematical operation of convolution. Illustrate your explanation with examples. Explain the meaning of the convolution theorem.

(iii) Sketch the derivation of Kirchhoff's scalar theory of diffraction which starts with Maxwell's equations of electromagnetism and results in a diffraction equation that has the same form as the Huygens-Fresnel diffraction equation.

(iv) Explain transverse (spatial) and longitudinal (temporal) coherence. Give examples. Show how these concepts are related to elementary quantum mechanics.

(v) Matrix methods are used to compute geometric optical effects in ray tracing programs. Explain how these methods work. Give specific examples.

(vi) Explain how a Fresnel zone plate focuses light. Describe the properties of zone plate lenses.

(vii) Prove one of the Fourier relations given in the transform pairs as shown schematically at the front of this exam paper.

(viii) Prove one of the Fourier theorems given in the information sheet at the front of this exam paper.

(ix) Explain how a diffraction grating works as a wavelength dispersive element. Show that the wavelength resolution of a diffraction grating is just dependent on the number of slits in the grating.

(x) Sketch the derivation of the van Cittert-Zernike theorem.

Question 3

The Huygens-Fresnel theory of diffraction is based on two principles:

1. each point on a wavefront acts as a source of secondary wavelets
2. these 2° wavelets produce a new wavefront by mutual interference.

(i) Draw a diagram to explain how the Huygens-Fresnel construction works.

This construction results in the Huygens-Fresnel diffraction equation:

$$\psi(\zeta,\eta) = \iint_{aperture} f(x,y)\frac{e^{ikR}}{R}\kappa(\chi)dxdy$$

where $f(x,y)$ is the transmission function of a diffraction aperture and $\psi(\zeta,\eta)$ is the diffracted field.

(ii) Explain the meaning of the function $\kappa(\chi)$ in the Huygens-Fresnel equation.

(iii) What is the physical significance of the term: $\dfrac{e^{ikR}}{R}$?

A more fundamental diffraction theory is the Kirchhoff Scalar theory, which is based on Maxwell's electromagnetic equations. In this theory, an expression for a field at point P, $\phi(P)$, arising from a diffraction aperture illuminated by a point source is given by the Kirchhoff-Fresnel equation:

$$\phi(P) = \frac{-i}{\lambda}\iint_{Aperture} \frac{Ae^{ik\rho}}{\rho}\frac{e^{ikr}}{r}\left(\frac{\hat{n}\bullet\hat{\rho}-\hat{n}\bullet\hat{r}}{2}\right)dS$$

(iv) Explain how the terms in the Huygens-Fresnel equation correspond to terms in the Kirchhoff-Fresnel equation.

(v) What is the physical meaning of the additional terms in the Kirchhoff-Fresnel equation that are absent in the Huygens-Fresnel equation?

In the case of far field diffraction, the diffraction equations can be used to show that the diffracted field is related to the aperture transmission function by a Fourier transform. This derivation is only valid when the Fraunhofer condition is satisfied:

$$\frac{r_{\max}^2}{2\lambda R_o} \ll 1$$

(vi) Explain the meaning of the terms in this equation.

(vii) What is the origin of this condition in terms of the derivation of the Fraunhofer diffraction equation?

(viii) Explain why this condition is not necessary when a lens is used to produce a diffracted field in its back focal plane.

Question 4

PART A

(i) How is the visibility of interference fringes defined so as to measure coherence properties of a thermal source? Illustrate your answer with a sketch.

(ii) Explain what is meant by the complex degree of coherence, γ_{12} or $\gamma(\mathbf{r}_1,\mathbf{r}_2,\tau)$?

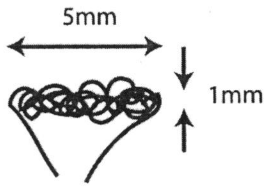
(iii) How is the complex degree of coherence related to the visibility of fringes?

(iv) Explain the van Cittert-Zernike theorem and how it allows one to compute the complex degree of coherence for a distant thermal light source (use words, diagrams and/or equations).

(v) Explain how you could experimentally determine the complex degree of coherence for the optical field produced by a distant thermal source.

PART B

The van Cittert-Zernike theorem can be used to compute the complex degree of coherence for distant thermal sources. Consider a car headlamp as a thermal source at a distance of 500m. Approximate the lamp filament as a rectangular source, 5mm long and 1mm wide.



(vi) Derive an expression for the complex degree of coherence of 500nm light produced by the car headlamp at a distance of 500m.

(vii) Plot the complex degree of coherence as a function of the separation between two points in the two principal directions (parallel to the two symmetry axes of the filament).

Two pinholes are placed in a screen 500 m from the car headlamp with a filter to remove all light except for that with a wavelength of 500 nm. The pinholes are oriented so that the line joining them is perpendicular to the long axis of the headlamp filament. The distance between the two pinholes can be varied. The interference pattern produced is observed on a second screen.

(viii) Describe the interference pattern observed as the distance between the pinholes is varied.

(ix) Sketch a series of interference patterns to illustrate key features of the complex degree of coherence.

END OF EXAM