THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS
FINAL EXAMINATION

PHYS3050 – Nuclear Physics

PHYS3031 – Advanced Optics and Nuclear Physics, Paper 1

Session 2, 2012

1. Time allowed – 2 hours
2. Total number of questions – 5
3. Total marks available – 100
4. Answer ALL questions. If math presents a difficulty use physical arguments and plain English.
5. **Answer Part A (questions 1, 2, 3) in one booklet and Part B (questions 4, 5) in a separate booklet.**
6. QUESTIONS ARE NOT OF EQUAL VALUE.
   Marks available for each question are shown in the examination paper.
7. University-approved calculators may be used.
8. All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
9. This paper may be retained by the candidate.

Useful Formulae and Tables

Table of quark properties:

Quark type (flavour)	u	d	s	c
Baryon number B	1/3	1/3	1/3	1/3
Spin J	1/2	1/2	1/2	1/2
Charge Q (units of e)	+2/3	−1/3	−1/3	+2/3
Isospin T	1/2	1/2	0	0
Isospin projection T_z	+1/2	−1/2	0	0
Strangeness S	0	0	−1	0
Charm C	0	0	0	+1

Some useful formulae:

- Radial Schrödinger equation for a central potential, letting $\psi(r,\theta,\phi) = \frac{R_l(r)}{r} Y_{lm}(\theta,\phi)$:

$$\frac{d^2 R_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right) R_l(r) = 0 \ .$$

- Density of states formula:

$$dn = \frac{4\pi p^2}{(2\pi\hbar)^3}\, dp$$

- $E^2 = m^2c^4 + p^2c^2$

- Wavefunction of K-shell electron ($1s$ electron):

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}} \exp(-Zr/a_B), \qquad a_B = \frac{\hbar^2}{m_e e^2}$$

Particle properties:

	Q	J^P	B	T	S	C
p	1	$\frac{1}{2}^+$	1	$\frac{1}{2}$	0	0
n	0	$\frac{1}{2}^+$	1	$\frac{1}{2}$	0	0
π^+	1	0^-	0	1	0	0
π^o	0	0^-	0	1	0	0
π^-	-1	0^-	0	1	0	0
K^+	1	0^-	0	$\frac{1}{2}$	1	0
K^-	-1	0^-	0	$\frac{1}{2}$	-1	0
K^o	0	0^-	0	$\frac{1}{2}$	1	0
K^o_S	0	0^-	0	$\frac{1}{2}$		0
K^o_L	0	0^-	0	$\frac{1}{2}$		0
η	0	0^-	0	0	0	0
ρ^+	1	1^-	0	1	0	0
ρ^o	0	1^-	0	1	0	0
ρ^-	-1	1^-	0	1	0	0
ω	0	1^-	0	0	0	0
Λ^o	0	$\frac{1}{2}^+$	1	0	-1	0
Σ^-	-1	$\frac{1}{2}^+$	1	1	-1	0
Σ^o	0	$\frac{1}{2}^+$	1	1	-1	0
Σ^+	1	$\frac{1}{2}^+$	1	1	-1	0
Δ^-	-1	$\frac{3}{2}^+$	1	$\frac{3}{2}$	0	0
Δ^o	0	$\frac{3}{2}^+$	1	$\frac{3}{2}$	0	0
Δ^+	1	$\frac{3}{2}^+$	1	$\frac{3}{2}$	0	0
Δ^{++}	2	$\frac{3}{2}^+$	1	$\frac{3}{2}$	0	0
Ξ^o	0	$\frac{1}{2}^+$	1	$\frac{1}{2}$	-2	0
Ξ^-	-1	$\frac{1}{2}^+$	1	$\frac{1}{2}$	-2	0
Ω^-	-1	$\frac{3}{2}^+$	1	0	-3	0
J/ψ	0	1^-	0	0	0	0
D^+	1	0^-	0	$\frac{1}{2}$	0	1
D^-	-1	0^-	0	$\frac{1}{2}$	0	-1
D^o	0	0^-	0	$\frac{1}{2}$	0	1

Part A (answer in a separate booklet)

Question 1 (30 marks)
Nucleons and pions

(a) Calculate the Compton radius of the pion (fm)

$$r_\pi = \frac{1}{m_\pi}$$

remembering that $m_\pi \approx 140\,\text{MeV}$.

(b) Explain very briefly in simple physical terms the origin of the Yukawa interaction between two nucleons

$$U_Y(r) = -g^2 \frac{e^{-m_\pi r}}{r}$$

In particular

i. explain qualitatively why this interaction is attractive

ii. give a qualitative reason for the strong dependence of this interaction on the pion mass

iii. give an estimate for the critical separation δr (fm) between two nucleons below which the Yukawa interaction is prominent

(c) Using the uncertainty principle and the separation δr (which was found in Q1b)

i. give an estimate for the kinetic energy K of nucleons in nuclei, expressing it via the proton and pion masses (remember that $m_p \approx 940\,\text{MeV}$ and, as was mentioned, $m_\pi \approx 140\,\text{MeV}$).

ii. knowing K estimate the typical potential energy U of nucleons in nuclei

iii. derive from the found K an estimate for a velocity v of nucleons in nuclei

Question 2 (10 marks)
Parity

(a) Explain why the parity proves to be useful for description of nuclear properties.

(b) Present the parity for pions. Explain briefly the qualitative physical reasons which lead to this result.

(c) Consider the radiation of a photon by the excited nucleus. Assume that this process takes place as the $E1$ transition.

i. Find the relation between the parities of the initial P_i and final P_f states of the nucleus.

ii. Present restrictions on the initial J_i and final J_f total momenta of the nucleus.

Hint: Remember that the photon emitted via a $E1$ transition occupies the state with quantum numbers 1^-.

Question 3 (10 marks)
Isospin

(a) Explain briefly which property of nucleons and pions prompts describing them using the isotopic spin.

(b) Present the isotopic spin and its projection for

i. proton

ii. neutron

iii. each of the three pions

(c) Calculate the projection of the isotopic spin for the nucleus with the mass number A and atomic number Z.

(d) Calculate the isotopic spin and its projection for the deuteron. Hints:

- remember that in this case $S = 1$, while $L = 0$ or 2
- remember also that the symmetrical (antisymmetrical) spin-function describes a state of spin 1 (spin zero)
- similarly, symmetrical (antisymmetrical) isospin function describes a state of isospin 1 (isospin zero)
- keep in mind that the Fermi statistics need to be satisfied.

Part B (answer in a separate booklet)

Question 4 (25 marks)

Shell model

(a) Consider the oscillator model for nuclear self consistent potential. Representing the spherically symmetric potential as a combination of x, y, and z parabolic potentials, and using the known result for the energy levels of a one-dimensional oscillator $E_n = (n+1/2)\omega$, find

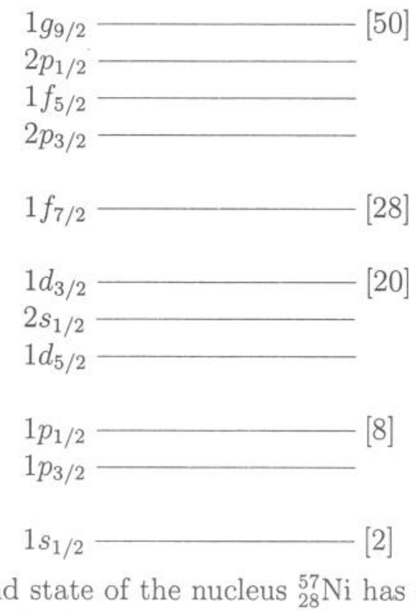
  i. The lowest four energy levels of the 3D oscillator

  ii. Parity of states in each shell

  iii. Capacity of each nuclear shell

  iv. Magic numbers of protons and neutrons

(b) With reference to the shell model diagram below, explain why magic numbers differ from the result of Q4a.



(c) The ground state of the nucleus $^{57}_{28}$Ni has quantum numbers $J^P = 3/2^-$.

  i. Using the diagram above, find the shell model configuration for the ground state.

  ii. The energy of the first two excitations are 769 keVand 1113 keV, with quantum numbers $5/2^-$ and $1/2^-$, respectively. What are the shell model configurations of these states?

  iii. By averaging the spin-orbital contribution to the Hamiltonian $H_{ls} = a(\boldsymbol{l} \cdot \boldsymbol{s})$ over the ground state and appropriate excited state, find the value of the spin-orbit constant a. Hence find the spin-orbit contribution to the energy of the ground state.

Question 5 (25 marks)

Beta decay

(a) For the following β-decays state whether the decay is of the Fermi type, Gamow-Teller type, both mechanisms contribute, or the decay is forbidden. Give the reasons.

$$\begin{aligned} n &\to p \\ {}^{111}_{51}\mathrm{Sb}\ (5/2^+) &\to {}^{111}_{50}\mathrm{Sn}\ (7/2^+) \\ {}^{20}_{9}\mathrm{F}\ (2^+) &\to {}^{20}_{10}\mathrm{Ne}\ (0^+) \\ {}^{152}_{63}\mathrm{Eu}\ (3^-) &\to {}^{152}_{62}\mathrm{Sm}^*\ (2^+) \\ {}^{14}_{8}\mathrm{O}\ (0^+) &\to {}^{14}_{7}\mathrm{N}^*\ (0^+) \\ {}^{3}\mathrm{H}\ (1/2^+) &\to {}^{3}\mathrm{He}\ (1/2^+) \\ {}^{171}_{68}\mathrm{Er}\ (5/2^-) &\to {}^{171}_{69}\mathrm{Tm}\ (1/2^+) \\ {}^{235}_{94}\mathrm{Pu}\ (5/2^+) &\to {}^{235}_{93}\mathrm{Np}\ (5/2^+) \end{aligned}$$

(b) The energy released in tritium β-decay is $\Delta E = m_{\mathrm{H}} - m_{\mathrm{He}} - m_e = 17\,\mathrm{keV} \ll m_e$. Using the Fermi golden rule

$$\lambda = 2\pi \left|V_{if}\right|^2 \rho_f, \qquad \rho_f = \frac{dn}{dE}$$

where V_{if} is the matrix element of the decay and ρ_f is the final phase space density. Assuming that the weak interaction decay matrix element is constant, derive an expression for the spectrum of β-electrons in tritium decay. Disregard corrections due to the Coulomb interaction in the final state.

(c) Give evidence that the weak interaction does not conserve parity.

— End of Exam —