THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS3050 – Nuclear Physics

Session 2, 2011

1. Time allowed – 2 hours
2. Total number of questions – 4
3. Total marks available – 60
4. Answer ALL questions
5. QUESTIONS ARE NOT OF EQUAL VALUE.
   Marks available for each question are shown in the examination paper.
6. University-approved calculators may be used.
7. All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
8. This paper may be retained by the candidate.

PHYS3050 — Useful Formulae and Tables

Table of quark properties:

Quark type (flavour)	u	d	s	c
Baryon number B	1/3	1/3	1/3	1/3
Spin J	1/2	1/2	1/2	1/2
Charge Q (units of e)	+2/3	−1/3	−1/3	+2/3
Isospin T	1/2	1/2	0	0
Isospin projection T_z	+1/2	−1/2	0	0
Strangeness S	0	0	−1	0
Charm C	0	0	0	+1

Some useful formulae:

- Radial Schrödinger equation for a central potential, letting $\psi(r,\theta,\phi) = \frac{R_l(r)}{r} Y_{lm}(\theta,\phi)$:

$$\frac{d^2R_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right) R_l(r) = 0 \ .$$

- Density of states formula:

$$dn = \frac{4\pi p^2}{(2\pi\hbar)^3}\, dp$$

- $E^2 = m^2c^4 + p^2c^2$

- Wavefunction of K-shell electron (1s electron):

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}} \exp(-Zr/a_B), \qquad a_B = \frac{\hbar^2}{m_e e^2}$$

Particle properties:

	Q	J^P	B	T	S	C
p	1	$1/2^+$	1	$1/2$	0	0
n	0	$1/2^+$	1	$1/2$	0	0
π^+	1	0^-	0	1	0	0
π^o	0	0^-	0	1	0	0
π^-	-1	0^-	0	1	0	0
K^+	1	0^-	0	$1/2$	1	0
K^-	-1	0^-	0	$1/2$	-1	0
K^o	0	0^-	0	$1/2$	1	0
K^o_S	0	0^-	0	$1/2$		0
K^o_L	0	0^-	0	$1/2$		0
η	0	0^-	0	0	0	0
ρ^+	1	1^-	0	1	0	0
ρ^o	0	1^-	0	1	0	0
ρ^-	-1	1^-	0	1	0	0
ω	0	1^-	0	0	0	0
Λ^o	0	$1/2^+$	1	0	-1	0
Σ^-	-1	$1/2^+$	1	1	-1	0
Σ^o	0	$1/2^+$	1	1	-1	0
Σ^+	1	$1/2^+$	1	1	-1	0
Δ^-	-1	$3/2^+$	1	$3/2$	0	0
Δ^o	0	$3/2^+$	1	$3/2$	0	0
Δ^+	1	$3/2^+$	1	$3/2$	0	0
Δ^{++}	2	$3/2^+$	1	$3/2$	0	0
Ξ^o	0	$1/2^+$	1	$1/2$	-2	0
Ξ^-	-1	$1/2^+$	1	$1/2$	-2	0
Ω^-	-1	$3/2^+$	1	0	-3	0
J/ψ	0	1^-	0	0	0	0
D^+	1	0^-	0	$1/2$	0	1
D^-	-1	0^-	0	$1/2$	0	-1
D^o	0	0^-	0	$1/2$	0	1

Question 1 (10 marks)

In heavy nuclei the number of neutrons is $N \approx 0.6A$ and the number of protons is $Z \approx 0.4A$. The nuclear radius is $R = r_0 A^{1/3}$, where $r \approx 1.1$ fm. Consider protons and neutrons in the Fermi gas approximation.

(a) Calculate values of the Fermi momentum p_F for protons and for neutrons (in MeV).

(b) Calculate values of the Fermi energy ϵ_F for protons and for neutrons (in MeV).

(c) Taking the binding energy of the last nucleon to be 8 MeV, estimate the depth of the potential for neutrons and protons. How does this compare with the typical depth of the nuclear self consistent potential, $V_0 \sim 50$ MeV? How can one account for the difference between the neutron and proton potentials?

Question 2 (15 marks)

The five lowest energy levels of ${}^{14}_{7}\text{N}$ are shown below. The quantum numbers J^P are indicated on the left hand side, and the energies relative to the ground state are shown on the right hand side, in MeV.

2^-	5.106
0^-	4.915
1^+	3.948
0^+	2.313
1^+	0.

(a) Using selection rules for electromagnetic transitions show all the decay channels which go via

i. E1-transition

ii. M1-transition

iii. E2-transition

(b) In natural units ($\hbar = c = 1$) the decay probabilities for E1 and M1 transitions are given by

$$W_{E1} = \frac{4}{3}\omega^3 \left|d_{fi}\right|^2 , \qquad W_{M1} = \frac{4}{3}\omega^3 \left|\mu_{fi}\right|^2 .$$

Using these equations estimate the lifetimes of 0^- and 0^+ states. For the estimate you can use the following typical values of the transition amplitudes:

$$d_{fi} \sim 0.1\, e \cdot \text{fm} , \qquad \mu_{fi} \sim \mu_N = \frac{e}{2m_p} .$$

Hint: You may use any units you like, but it is convenient to find value of the lifetime in inverse MeV using natural units and then convert the result into seconds using $\hbar = 6.582 \times 10^{-22}\, \text{MeV} \cdot s$

Question 3 (15 marks)

(a) For the following β-decays state whether the decay will be Fermi type, Gamow-Teller type, both, or forbidden. Give the reasons.

$$
\begin{aligned}
n &\rightarrow p \\
{}^{111}_{51}\mathrm{Sb}\ (5/2^+) &\rightarrow {}^{111}_{50}\mathrm{Sn}\ (7/2^+) \\
{}^{3}\mathrm{H}\ (1/2^+) &\rightarrow {}^{3}\mathrm{He}\ (1/2^+) \\
{}^{171}_{68}\mathrm{Er}\ (5/2^-) &\rightarrow {}^{171}_{69}\mathrm{Tm}\ (1/2^+) \\
{}^{235}_{94}\mathrm{Pu}\ (5/2^+) &\rightarrow {}^{235}_{93}\mathrm{Np}\ (5/2^+)
\end{aligned}
$$

(b) K-electron capture

$$(Z, N) + e^- \rightarrow (Z-1, N+1) + \nu$$

is described by the Fermi theory of weak interaction. Using the Fermi golden rule

$$\frac{dW}{dt} = 2\pi \left|V_{if}\right|^2 \rho_f \,,$$

where

$$\rho_f = \int \delta(\omega - p) \frac{dn}{d\omega}$$

is the density of states of the emitted neutrino, find the dependence of the electron-capture probability on

i. the nuclear charge Z,

ii. the released energy ω – the energy of the escaping neutrino.

(c) Explain why K-shell capture competes with positron emission for large Z.

Question 4 (20 marks)

Use the table of particle properties provided to

(a) Give the quark assignments for the following particles:

$$\text{p, n, } \pi^+,\ \Lambda^o,\ \Omega^-,\ \Sigma^+,\ D^o.$$

(b) Using Δ^{++} as an example, explain why the extra quantum number, colour, is necessary.

(c) Plot, on appropriate axes, the Gellman-Ne'eman eightfold diagram for the meson family (K^+, K^o, π^-, π^o, π^+, η, η', K^-, $\bar{K}^o$)

(d) State whether the following reactions proceed via the strong, weak, or electromagnetic interactions, or are forbidden. Present corresponding Feynman diagrams for allowed reactions at the quark-lepton level (with intermediate particles g, γ, W, Z).

$$\rho^+ \to \pi^+ + \pi^o$$
$$\Lambda^o \to p + \mu^- + \bar{\nu}_\mu$$
$$\Sigma^+ \to \Lambda^o + e^+ + \nu_e$$
$$K^+ + n \to \Sigma^+ + \pi^o$$
$$\pi^- \to e^- + \gamma$$
$$\bar{\nu}_e + p \to n + e^+$$
$$\pi^- \to e^- + \gamma$$
$$\pi^o \to 2\gamma$$
$$\Delta^o \to \pi^- + p$$
$$\text{J}/\psi \to \mu^+ + \mu^-$$

— End of Exam —