Final test, 2002

Nuclear physics 3050

All questions are of equal value

Question 1

A particle of mass μ is scattering on the central potential

$$V(r) = \begin{cases} -V_0 & r \leq R \\ 0 & r > R \end{cases}$$

which models the n-p interaction. The scattering length a is defined as $a = -\lim_{k\to 0} \delta/k$, where δ is the scattering phase shift. You may use without proof the relation $\sigma = 4\pi a^2$, where σ is scattering cross section.

(a). Write down continuity equations for the s-wave state with negative energy (bound state) and for the s-wave state with positive energy (scattering).

(b). Assuming that there is an s-wave bound state with very small binding energy ($E_0 \ll V_0$, $\sqrt{2\mu E_0} \cdot R \ll 1$) demonstrate that the scattering length is equal to

$$a = \frac{1}{\sqrt{2\mu E_0}}.$$

Hence demonstrate that the low energy scattering cross section is

$$\sigma = \frac{2\pi}{\mu}\frac{1}{E_0}.$$

The units correspond to $\hbar = c = 1$.

(c). The deuteron has spin $S = 1$. Binding energy of the deuteron is $E_0 = 2.2 MeV$ and the nucleon mass is $m \approx 940 MeV$. Using the above formula for σ, calculate value of the triplet ($S = 1$) n - p scattering cross section at zero energy. Express the result in barns (1b=$10^{-24} cm^2$, $1 fm = 10^{-13} cm \approx 1/197 MeV$).

Question 2

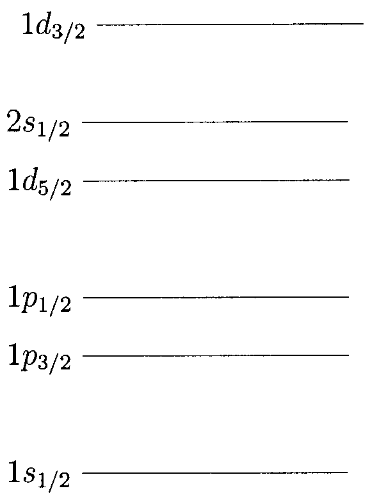
(a). Starting from the operator of the magnetic moment $\mu = \mu_N(g_l \mathbf{l} + g_s \mathbf{s})$, derive the shell model formulae for the magnetic moment of an even-odd nucleus

$$\mu = \mu_N \left[g_l \left(j - \tfrac{1}{2} \right) + \tfrac{1}{2} g_s \right] \quad \text{if } j = l + \tfrac{1}{2}$$
$$\mu = \mu_N \left[g_l \tfrac{j(j+3/2)}{j+1} - \tfrac{1}{2} \tfrac{j}{j+1} g_s \right] \quad \text{if } j = l - \tfrac{1}{2},$$

where $\mathbf{j} = \mathbf{l} + \mathbf{s}$ is the angular momentum of the external nucleon which coincides with the nuclear spin I. $g_l = 1$, $g_s \approx 5.6 \times 0.6$ for proton and $g_l = 0$, $g_s \approx -3.8 \times 0.6$ for neutron.

(b). **(i)** Using shell model energy levels presented in the picture, and results of part **(a)**, calculate the ground state spin I, parity and magnetic moment for $^{26}_{12}$Mg, and $^{27}_{13}$Al nuclei.

(ii) Answer the same question but for the first excited state of $^{27}_{13}$Al.



(c). The nucleus $^{238}_{94}$Pu has a static quadrupole deformation. Explain the nature of the lowest excited states in this nucleus. Find the angular momentum and parity of the ground state, first excited state, and second excited state.

Question 3

(a). For the following β-decays state whether the decay is of the Fermi type, Gamow-Teller type, both mechanisms contribute, or the decay is forbidden. Give the reasons.

$$\mathrm{n} \rightarrow \mathrm{p}$$
$${}^{14}_{8}\mathrm{O}(0^+) \rightarrow {}^{14}_{7}\mathrm{N}^*(0^+)$$
$${}^{6}_{2}\mathrm{He}(0^+) \rightarrow {}^{6}_{3}\mathrm{Li}(1^+)$$
$${}^{17}_{7}\mathrm{N}(1/2^-) \rightarrow {}^{17}_{8}\mathrm{O}(5/2^+)$$
$${}^{13}_{5}\mathrm{B}(3/2^-) \rightarrow {}^{13}_{6}\mathrm{C}(1/2^-)$$
$${}^{111}_{50}\mathrm{Sn}(7/2^+) \rightarrow {}^{111}_{49}\mathrm{In}(9/2^+)$$
$${}^{3}_{1}\mathrm{H}(1/2^+) \rightarrow {}^{3}_{2}\mathrm{He}(1/2^+)$$

The wave function of the K-electron (1s-electron) is

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}} \exp(-Zr/a_B).$$

The K-electron capture

$$(Z, N) + e^- \rightarrow (Z-1, N+1) + \nu$$

is described by the Fermi theory of weak interaction. The golden rule for this process reads

$$\frac{dW}{dt} = 2\pi |V_{fi}|^2 \rho_\nu,$$

where

$$\rho_\nu = \int \delta(\omega - p) \frac{d^3p}{(2\pi)^3} = \frac{\omega^2}{2\pi^2}$$

is the density of states of the emitted neutrino,

(b). Find the dependence of the probability of the K-electron (K=1s) capture on the nuclear charge Z.

(c). Find also the dependence of the probability of the K-electron capture on the released energy (i.e. on the energy of the escaping neutrino).

Question 4

(a). Give the quark content of the following particles: p, n, π^+, Λ, Ω^-, J/ψ, D^0.

Quantum numbers of these particles are presented in the table. (Q is electric charge, B is barion number, T is isospin, S is strangeness, and C is charm)

particle	Q	B	T	S	C
p	1	1	1/2	0	0
n	0	1	1/2	0	0
π^+	1	0	1	0	0
Λ	0	1	0	-1	0
Ω^-	-1	1	0	-3	0
J/ψ	0	0	0	0	0
D^0	0	0	1/2	0	1

(b). Using Ω^- as an example, explain why an extra quantum number, colour, is necessary.

(c). State whether the following reactions proceed via strong, electromagnetic, or weak interactions, or are forbidden. Present corresponding Feynman diagrams for these reactions at the quark-lepton level (with intermediate particles g, γ, W, Z)

$$p + \bar{\nu}_e \to n + e^+$$
$$\rho^+ \to \pi^+ + \pi^0$$
$$\pi^- \to e^- + \gamma$$
$$\pi^0 \to 2\gamma$$
$$\Lambda \to p + \pi^-$$
$$\pi^+ \to \mu^+ + \nu_\mu$$
$$J/\psi \to \mu^+ + \mu^-$$