THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION

NOVEMBER 2013

**PHYS3060/3031** p2.

**ADVANCED OPTICS**

Time Allowed – 2 Hours

Total number of questions – 4

Answer ALL questions

Questions are of **EQUAL** value

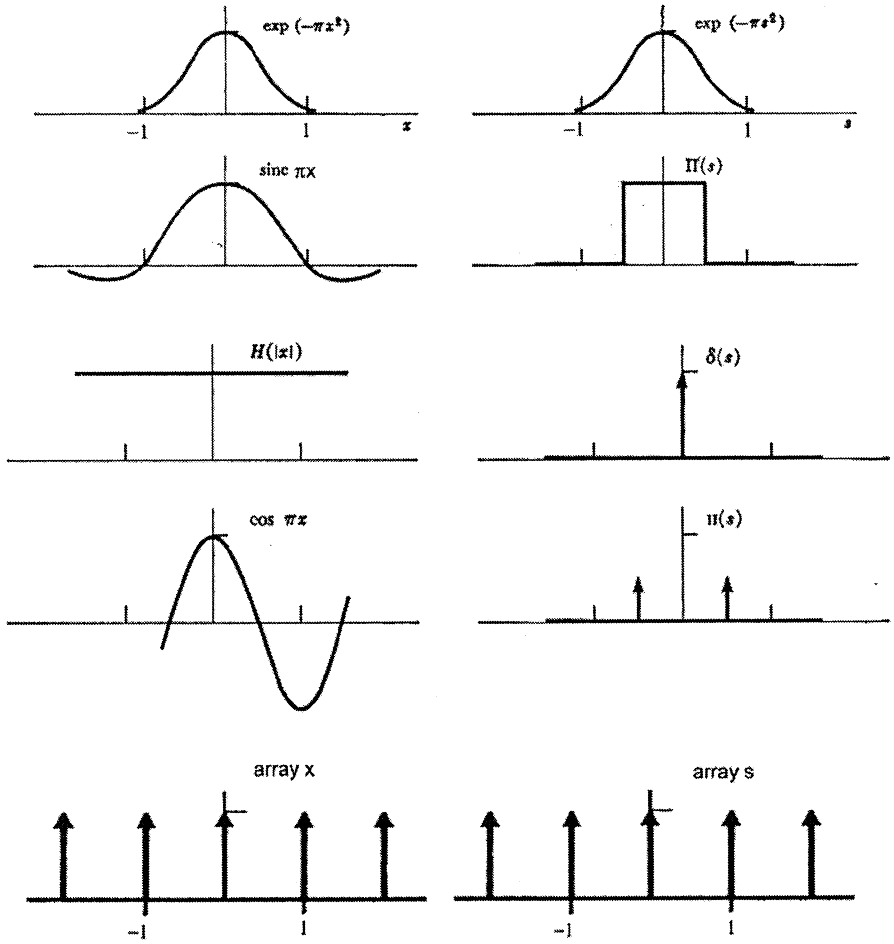
Candidates should provide their own university approved calculator

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

Candidates may keep the examination script.

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-2\pi ixs} dx, \quad f(x) = \int_{-\infty}^{\infty} F(s) e^{2\pi ixs} ds,$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$




| Theorem | $f(x)$ | $F(s)$ |
|---|---|---|
| Similarity | $f(ax)$ | $\frac{1}{\lvert a \rvert} F(s)$ |
| Linearity | $\alpha f(x) + \beta g(x)$ | $\alpha F(s) + \beta F(s)$ |
| Shift | $f(x-a)$ | $e^{-i2\pi as} F(s)$ |
| Convolution | $f(x) \otimes g(x)$ | $F(s)G(s)$ |

## QUESTION 1

Consider a grating function with a sinusoidal transmission profile (see diagram below). The period of the oscillations is 0.1mm and there are a total of 12 periods within the aperture.


i. Write down an expression for the aperture function.

ii. Calculate the viewing distance that will satisfy the far-field diffraction condition for illumination with visible light of $\lambda = 550\text{nm}$.

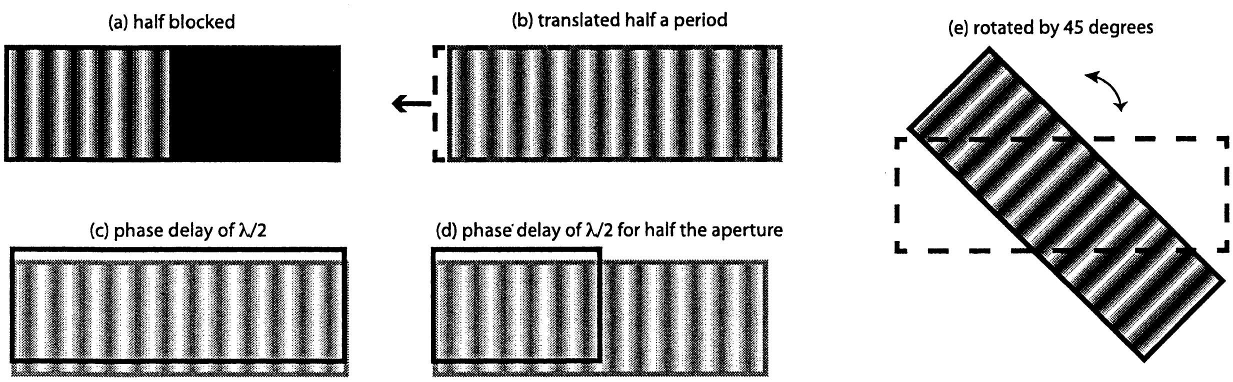
iii. Derive the resulting far-field diffraction pattern (field amplitude and intensity)

iv. Make a plot of the observed diffracted intensity pattern as a function of diffraction angle marking the important features.

v. Comment on the difference between this and an "array of slits" type diffraction grating.

vi. Use the Fourier theorems to describe qualitatively what will happen to the diffraction pattern in the following cases:

   a. Half of the aperture is blocked
   b. The object is translated by ½ a grating period.
   c. A phase delay equivalent to half a wavelength is applied to the aperture function.
   d. The phase delay in (c) is applied to half of the aperture.
   e. The aperture function is rotated by 45 degrees



## QUESTION 2

### PART A

With the aid of equations and diagrams how Zernike's phase contrast imaging works. Comment on the approximations used in the formulation and practical aspects of implementing this technique for phase contrast microscopy.

### PART B

A transparent piece of plastic has been embossed in such a way that it encodes the Fourier transform of a crocodile. When this transparency is placed in the object plane of the optical system, shown in Figure 1., the image of the crocodile appears in the focal plane.

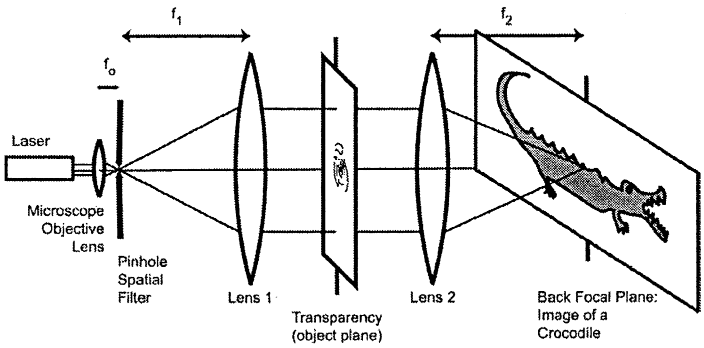



FIGURE 1

You are given two such transparencies, each encoding the Fourier transform of a crocodile. You place both of them side-by-side in the object plane of the above optical system so that their origins are slightly separated in a direction perpendicular to the axis of the optical system, as in Figure 2. Both transparencies are fully illuminated by the coherent optical beam. Using your knowledge of Fourier theorems and diffraction describe the image produced by this arrangement of two transparencies.

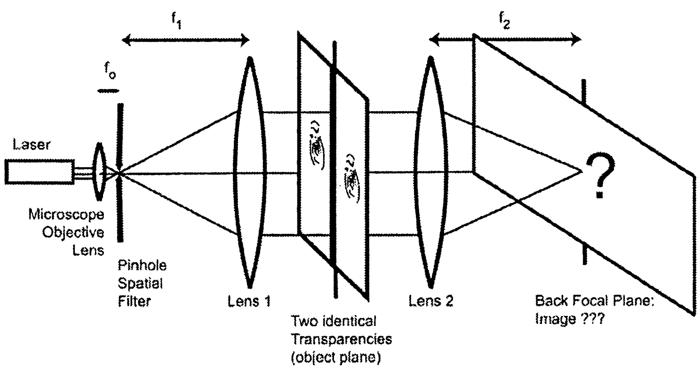



FIGURE 2

## QUESTION 3

(i) Explain how a Fresnel zone plate acts as a lens?
(ii) Derive the following equation for the path difference for the m$^{th}$ Fresnel zone:

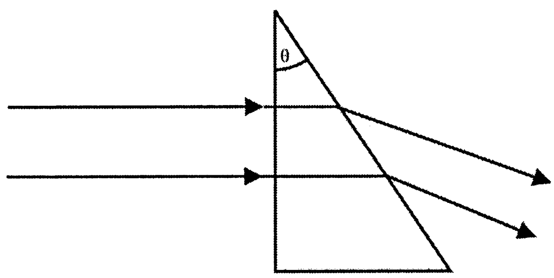
$$\frac{m\lambda}{2} = \frac{{r_m}^2}{2}\left(\frac{1}{r_0} + \frac{1}{b_0}\right)$$

Design a Fresnel lens with a focal length of 45 μm for a 980 nm laser (assume the source is collimated).

(iii) What is the radius of the first three Fresnel zones for this lens?
(iv) Calculate the positions of two other focal lengths produced by this lens.
(v) What is the benefit of adding extra zones to the Fresnel zone plates?
(vi) What is the limit to adding extra zones?

Fresnel zone plates are used routinely in adaptive optics to create lenses with dynamically configurable focal lengths. Similar zone plate-type constructions can be used to mimic "almost" any optical element.

(vii) Can you think of a way to use a similar construction to create the equivalent of a refracting prism (refractive index n =1.45) in the following configuration:

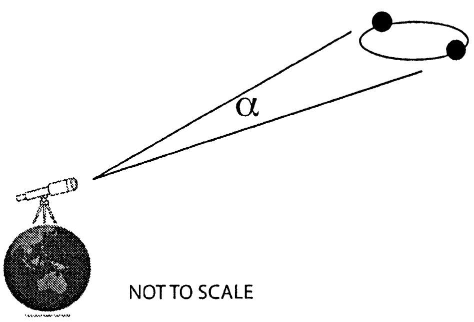


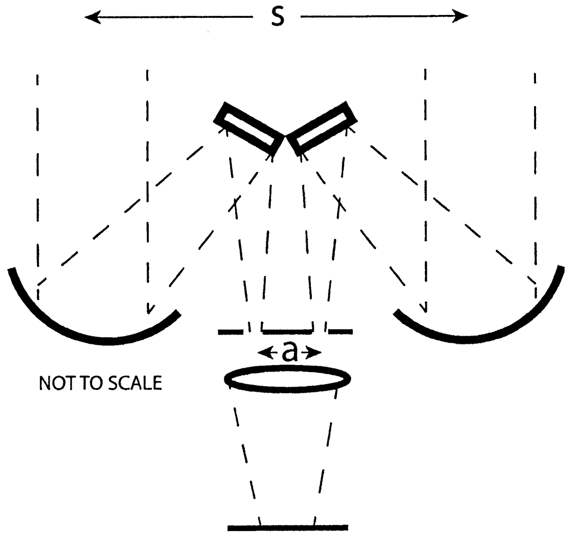
## QUESTION 4

The following formula can be used to describe the intensity pattern produced by two pinholes illuminated with a monochromatic light source of finite coherence length.

$$\langle I(P)\rangle = |K_1|^2\langle I_1\rangle + |K_2|^2\langle I_2\rangle + \left[\langle I_1\rangle^{1/2}\langle I_2\rangle^{1/2}\right]|\gamma_{12}|\cos\left(\alpha(r_1,r_2,\tau)-\delta\right)$$

(i) Explain the physical origin of each of the terms and their effect on the observed interference pattern.




Alpha Newburi is a binary star system consisting of two stars of roughly equal brightness orbiting around their common centre of mass. Alpha Newburi is located at a distance of 4.37 light years from Earth and the separation between the two stars subtends an angle, α, of 2.2 arc-seconds on the night's sky; the individual stars can be considered as point sources. (1 light year = $9.46\times10^{15}$m)

(ii) Assuming that the light from the stars is quasi-monochromatic (λ=550nm) derive an expression for the complex degree of coherence from the binary star.

A stellar interferometer (see diagram) is used to measure the angular separation between the two stars. It consists of two large collecting mirrors separated by a distance, s, which focuses light onto two separate pinholes separated by a distance, a - light from the two pinholes is then allowed to interfere at an observation plane in the far-field.

(iii) What is the minimum separation between the collecting mirrors, s, that will provide a minimum in fringe visibility for Alpha Newburi?

(iv) Comment on the influence of the separation between the two pinholes, a.

(v) How will the observed diffraction pattern change as the two stars orbit around their centre of mass?