THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS3050 – Nuclear Physics

PHYS3031 – Advanced Optics and Nuclear Physics, Paper 1

Session 2, 2013

1. Time allowed – 2 hours
2. Total number of questions – 4
3. Total marks available – 100
4. Answer ALL questions. If math presents a difficulty use physical arguments and plain English.
5. **Answer Part A (questions 1, 2) in one booklet and Part B (questions 3, 4) in a separate booklet.**
6. QUESTIONS ARE NOT OF EQUAL VALUE.
   Marks available for each question are shown in the examination paper.
7. University-approved calculators may be used.
8. All answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.
9. This paper may be retained by the candidate.

Useful Formulae

- Radial Schrödinger equation for a central potential, letting $\psi(r,\theta,\phi) = \frac{R_l(r)}{r} Y_{lm}(\theta,\phi)$:

$$\frac{d^2R_l(r)}{dr^2} + \frac{2m}{\hbar^2}\left(E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right) R_l(r) = 0 \; .$$

- Density of states formula:

$$dn = \frac{4\pi p^2}{(2\pi\hbar)^3}\, dp$$

- $E^2 = m^2c^4 + p^2c^2$

- Wavefunction of K-shell electron ($1s$ electron):

$$\psi(r) = \sqrt{\frac{Z^3}{\pi a_B^3}}\exp(-Zr/a_B), \qquad a_B = \frac{\hbar^2}{m_e e^2}$$

Part A (answer in a separate booklet)

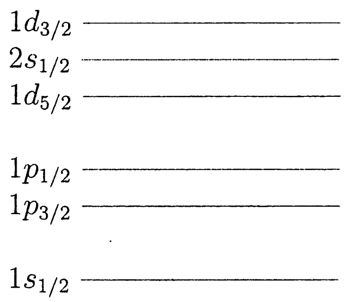
Question 1 (25 marks)

(a) Starting from the operator of the magnetic moment $\boldsymbol{\mu} = \mu_N(g_l \boldsymbol{l} + g_s \boldsymbol{s})$, derive the shell model formulae for the magnetic moment of an even-odd nucleus

$$\mu = \begin{cases} \mu_N \left[g_l \left(j - \frac{1}{2} \right) + \frac{1}{2} g_s \right] & \text{if } j = l + \frac{1}{2} \\ \mu_N \left[g_l \frac{j(j+\frac{3}{2})}{j+1} - \frac{j}{2(j+1)} g_s \right] & \text{if } j = l - \frac{1}{2} \end{cases}$$

where $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$ is the angular momentum of the unpaired nucleon. For protons $g_l = 1$, $g_s = 5.6$ and for neutrons $g_l = 0$, $g_s = -3.8$.

(b) i. Use the shell model shown below to compute the ground state angular momentum, isospin projection, parity, and magnetic moment of the ${}^{11}_{6}\text{C}$, ${}^{12}_{6}\text{C}$, and ${}^{13}_{6}\text{C}$ nuclei.



ii. What are the angular momentum, isospin projection, and parity for the excited ${}^{11}_{6}\text{C}$ and ${}^{13}_{6}\text{C}$ nuclei with an unpaired nucleon in the first excited state.

iii. In natural units ($\hbar = c = 1$) the decay probabilities for E1 and M1 transitions are given by

$$w_{E1} = \frac{4}{3}\omega^3 \left| d_{fi} \right|^2 , \qquad w_{M1} = \frac{4}{3}\omega^3 \left| \mu_{fi} \right|^2 .$$

Using these equations estimate the lifetimes of the first excited states of ${}^{11}_{6}\text{C}$ and ${}^{13}_{6}\text{C}$. The transition energies are 2.0 MeV and 3.1 MeV, respectively. You can use the following typical values of the transition amplitudes:

$$d_{fi} \sim 0.1\, e \cdot \text{fm} , \qquad \mu_{fi} \sim \mu_N = \frac{e}{2m_p} .$$

Hint: You may use any units you like, but it is convenient to find value of the lifetime in inverse MeV using natural units and then convert the result into seconds using $\hbar = 6.6 \times 10^{-22}$ MeV $\cdot\, s$

Question 2 (25 marks)

(a) For the following β-decays state whether the decay is of the Fermi type, Gamow-Teller type, both mechanisms contribute, or the decay is forbidden. Give the reasons.

$$\begin{aligned}
n &\rightarrow p \\
{}^{111}_{51}\mathrm{Sb}\ (5/2^+) &\rightarrow {}^{111}_{50}\mathrm{Sn}\ (7/2^+) \\
{}^{20}_{9}\mathrm{F}\ (2^+) &\rightarrow {}^{20}_{10}\mathrm{Ne}\ (0^+) \\
{}^{152}_{63}\mathrm{Eu}\ (3^-) &\rightarrow {}^{152}_{62}\mathrm{Sm}^*\ (2^+) \\
{}^{14}_{8}\mathrm{O}\ (0^+) &\rightarrow {}^{14}_{7}\mathrm{N}^*\ (0^+) \\
{}^{3}\mathrm{H}\ (1/2^+) &\rightarrow {}^{3}\mathrm{He}\ (1/2^+) \\
{}^{171}_{68}\mathrm{Er}\ (5/2^-) &\rightarrow {}^{171}_{69}\mathrm{Tm}\ (1/2^+) \\
{}^{235}_{94}\mathrm{Pu}\ (5/2^+) &\rightarrow {}^{235}_{93}\mathrm{Np}\ (5/2^+)
\end{aligned}$$

(b) The energy released in β–decay $A' \rightarrow A + e^- + \tilde{\nu}_e$ is $\Delta E = Q - m_e$ where $Q = m'_A - m_A$. Derive an expression for the spectrum of β-electrons using the Fermi golden rule

$$\lambda = 2\pi \left|V_{if}\right|^2 \rho_f, \qquad \rho_f = \frac{dn}{dE}$$

where V_{if} is the matrix element of the decay and ρ_f is the final phase space density. You may assume that the weak interaction decay matrix element is constant for all energies of escaping electrons, ε. In this part, disregard corrections due to the Coulomb interaction in the final state.

(c) The formula we just derived should be corrected due to the Coulomb interaction between the emitted electron and the nucleus. This is achieved by multiplying the derived spectrum by the 'Sommerfeld factor' $F(Z, \varepsilon)$. Is F greater than or less than one for β^- decay? Explain your answer.

(d) Kurie defined a function

$$K(\varepsilon) = \left[\frac{dw/d\varepsilon}{F(Z,\varepsilon)\varepsilon\sqrt{\varepsilon^2 - m_e^2}}\right]^{1/2}.$$

Plot K against ε. Thus explain how the β-decay spectrum can be used to measure the anti-neutrino mass.

Part B (answer in a separate booklet)

Nuclear units (fm, MeV, for distances and energies) are adopted: $\hbar c = 197$ MeV · fm. Please be brief, only the most necessary facts and topics should be discussed.

Question 3 (30 marks)
Sub-nuclear structure, quarks, parity, etc.

(a) Estimate the typical inter-nucleon separation (fm) via fundamental physical parameters. Estimate also the typical kinetic and potential energies of nucleons in nuclei (MeV), and give an estimate for a typical velocity of nucleons. Compare these velocities with the velocities of electrons in atoms.

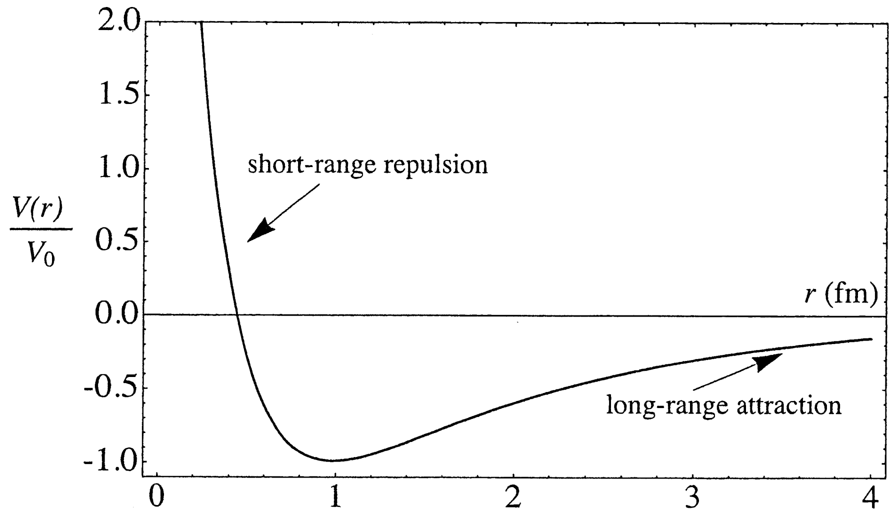


Figure 1: Nuclear potential $V(r)$ (arb. units) versus nuclear separation r (fm).

(b) Consider Fig. 1, which shows a sketch for the potential responsible for the nucleon-nucleon interaction (for the present semi-quantitative discussion it suffices to consider only the scalar part of this potential shown in Fig. 1, neglecting the spin and isospin dependence).

i. Explain qualitatively the role, which π-, ρ-, and ω-mesons play in creation of the potential in Fig. 1. Indicate which of them are responsible for attraction and which ones produce repulsion. Explain also how the attractive and repulsive nature of the nuclear forces is related to the spins of the mesons in question.

ii. Using Fig. 1 estimate the mass of the meson which is responsible for the attraction between nucleons. *Hint:* to facilitate calculations please note that Fig. 1 gives the ratio $V(3)/V(4) \approx 2$ (the necessary numerical calculations do not require calculators at all if one also keeps in mind a simple fact that $\ln 2 \approx 0.7$).

iii. Using Fig. 1 give a rough estimate of the masses of mesons which are responsible for the repulsive part of the potential. (Note the difference between parts ii. and iii.: the light meson mass can be derived from Fig. 1 accurately (say, 30% accuracy), while only a rough estimate can be made in relation to the masses of heavy mesons.)

iv. Using the potential in Fig. 1 present arguments in favour of the Fermi-gas model for nuclei and liquid-nuclear model.

(c) Using the Fermi-gas model calculate the averaged kinetic energy of each nucleon in nuclei (MeV) assuming that $N \approx Z = A/2$. Compare this result with the estimate from Part (a).
Hint: Remember that in the Fermi model the only two dimensional parameters are $r_0 = 1.1$ fm and $m_p = 938 \approx 1000$ MeV. Thus, if you have difficulties with derivations, numerical coefficients, etc, you may derive the necessary result from simple dimensional arguments (a numerical coefficient, which cannot be derived this way, is not important for qualitative description).

(d) Outline briefly the known, important experimental facts, which indicate that the nuclear interaction incorporates the spin- and isospin-dependent structure.

Question 4 (20 marks)
Sub-nuclear structure, quarks, parity, etc.

(a) Present the masses and charges for the quarks of the first generation, i. e. for u and d quarks.

(b) Present the quark content for the proton, neutron, triplet of pions π^0, $\pi^\pm$, triplet of ρ- mesons ρ^0, $\rho^\pm$, and ω-meson.

(c) Give a definition for the parity P.

(d) Present the quantum numbers J^P for the proton, neutron, pions, ρ and ω mesons.

(e) Outline in simple physical terms the fundamental reason, which prompts the parity to be violated in the Weak Interaction.

(f) Give an estimate for the effective radius of the Weak Interaction via the mass of the $W^\pm$ (or Z) boson; compare it with the size of the proton. Explain in simple physical terms the reason that usually prevents the Weak Interaction from manifesting itself strongly in nuclear and atomic physics. Propose an experiment in which the Weak Interaction would manifest itself strongly.

— End of Exam —