## QUESTION 1 (7 marks)

(a) Using the dilute gas approximation $N_j << g_j$ for all $j$, derive the thermodynamic weight for the Maxwell-Boltzmann statistics from the Fermi-Dirac and Bose-Einstein results.

(b) By comparing the Maxwell-Boltzmann thermodynamic weight with the Boltzmann thermodynamics weight find the Maxwell-Boltzmann distribution.

(c) If $N_j/g_j = (N/Z)\exp(-\varepsilon_j/kT)$, show that the Helmholtz function is given by

$$F = U - TS = NkT\left\{\ln\left(\frac{N}{Z}\right) - 1\right\}$$

(d) The partition function of a system that obeys Maxwell-Boltzmann statistics is given by $Z = bVT^{\alpha}$, where $\alpha$ and $b$ are constants. Calculate $U$, $P$ and $S$, and find the relationship between the pressure $P$ and internal energy density $U/V$.

$-S$

## QUESTION 2 (8 marks)

(a) The Gibbs ensemble is a collection of identical systems with different initial conditions all in contact with the same reservoir. If the probability of a system being in state $j$ is $p_j$ then in the ensemble $n_j = p_j n$ systems are in state $j$. If the thermodynamic weight for the ensemble is

$$w_n = \frac{n!}{n_1!n_2!\ldots n_j!\ldots}$$

derive the Gibbs entropy.

$H = \frac{p^2}{2m} + \Phi(q)$

(b) If $p_r = \exp(-\beta E_r)/Z(\beta)$ derive the Helmholtz function.

(c) If $H(q,p)$ is the Hamiltonian for the ideal gas, evaluate the partition function $Z(\beta)$ where

$$Z = \int \exp(-\beta H(q,p))dqdp$$

(d) Find the internal energy and heat capacity for a classical ideal gas from

$$U = kT^2\frac{\partial}{\partial T}\ln Z.$$

$F = U - TS$

(e) If the internal energy is given by

$$\langle U\rangle = \frac{1}{Z}\int H(q,p)\exp(-\beta H(q,p))dqdp$$

show that

$\ln(1)$

$$\frac{\partial^2}{\partial\beta^2}\ln Z = kT^2C_V$$

and

$\ln(n!) = n\ln(n)$

$$\frac{\partial^2}{\partial\beta^2}\ln Z = \langle U^2\rangle - \langle U\rangle^2.$$

$\ln\left(\frac{n}{n}\right)$