THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

PHYS2120
COMPUTATIONAL PHYSICS PAPER

FINAL EXAM

SESSION 1 2013

Answer all questions

Time allowed = 2 hours

Total number of questions = 6

Marks = 45

This paper forms **20%** of the total assessment for **PHYS2120**

The questions are **NOT** of equal value.

This paper may be retained by the candidate.

Students must supply their own UNSW approved calculator.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

**Question 1 (8 marks)**

(a) Describe briefly how Monte-Carlo integration can be used to evaluate an integral.

(b) Explain how you would generate random numbers in the C programming language, including the methods you could use to seed these random numbers. You do not need to write any code.

(c) Write a brief pseudocode or flow chart algorithm for finding the volume of a sphere using Monte Carlo integration. You should include a clear description of what you are doing at each step.

**Question 2 (7 marks)**

(a) What are rounding or round-off errors in the context of a computer program?

(b) What is one simple thing you can do in C to minimize round-off errors?

(c) Explain why round-off errors are not as big an issue for computer programs now, in 2013, as they were 30 years ago.

(d) Round-off errors are part of a larger class of errors, which are particularly relevant to the digitization of data. What is the larger class of error termed?

(e) For the larger class of error described in (d), discuss how much uncertainty this error contributes to the general uncertainly in the calculations if the signal you are measuring is much larger than the least significant bit (LSB) of your floating point decimal storage area in memory in your computer/ program? In your answer, explain what the least significant bit actually is, and what you would expect the mean and rms of this error to be, if the signal is >> LSB. Assume the last digit is rounded up rather than truncated during calculations.

**Question 3 (7 marks)**

First order ordinary differential equations (ODEs) can be solved using Euler's method. Given an initial condition ($x_0$, $y_0$), successive points on the solution curve (x, y(x)) to the ODE can be generated by taking equal steps of size $h$ in the independent variable $x$, and determining the new $y$ value using $y_{i+1} = y_i + h(f(x_i, y_i))$. The numerical solution is then a set of points that approximate the solution curve.

(a) Derive an expression for the simple Euler method using a Taylor series expansion. Show all steps in your derivation.

(b) It is common in physics for the second derivative of a particular quantity to be known, rather than the first derivative. A good example of this is that Newton's second law tells us that

$$\frac{d^2y}{dt^2} = \frac{F}{m}$$

where $y$ is the displacement of an object with time, $t$, $F$ is the applied force, and $m$ is the mass of the object.

Explain how you would go about manipulating the above differential equation so that the problem can be solved using the simple Euler method. Your answer should include all working, and should produce two equations which could then be used in a program to estimate y at any time t. Make sure you explain clearly your logic at each step.

**Question 4 (9 marks)**

You are given a data set containing measurements of the number of radioactive particles present in a sample of material measured with time elapsed in days. The sample of material contains a fraction of an unknown radioactive material, $^nX$; (assume only one type of radioactive nucleus is present, and that all other material is radioactively inert). You are asked to predict how many of the radioactive nuclei will be present after a time t, which is extrapolated quite far in the future (3000 days) from the times measured in the dataset you are presented with.

The measurements you are given are

| N(t) | t (days) |
|---|---|
| 3.000E+08 | 0 |
| 1.820E+08 | 50 |
| 1.104E+08 | 100 |
| 6.694E+07 | 150 |
| 4.060E+07 | 200 |
| 2.463E+07 | 250 |

The exact, analytical relationship for radioactive nuclear decay is

$$N_X = N_X(0)e^{\frac{-t}{\tau}},$$

where $N_X$ is the number of *radioactive* nuclei of the unknown substance, $^nX$, remaining in the sample after time $t$ has elapsed, $N_X(0)$ is the number of nuclei of the unknown substance initially present when the experiment started, and $\tau$ is the "time constant" for the decay.

i) Using the dataset above, to work out an estimate of the time constant $\tau$ for this dataset. Discuss the best way to estimate $\tau$ from this data, and give a rough estimate the uncertainty that $\tau$ will have.

ii) Use a first order Taylor series approximation for the above equation to derive a relationship between $N_x$ and $t$ that can be compared with the measured data.

iii) Discuss how you will use the numerical approximation from (ii) to predict the number of nuclei of $^nX$ that will be present in the sample after 3000 days.

iv) Discuss how the accuracy of your numerical approximation will be dependent on the original data set you were given to solve the problem, particularly in terms of the time interval of the measurements.

**Question 5 (7 marks)**

You are given a script in a language based on C (e.g. perl, IDL, python) which can read in a FITS image of data from the NASA moderate resolution imaging spectroradiometer satellite (MODIS), which provides infrared imaging of every Earth volcano on about a time interval of every two days. You are given images at the infrared wavelengths of both 10 and 12 microns. The images are in two fits files, one for each frequency.

(a) What does FITS stand for? Briefly explain the rationale behind the FITS image format.
(b) Describe, qualitatively and briefly, what the header and data sections in a FITS image are. How does the header component contribute to the rationale you have given in (a).
(c) Explain what is meant by a "pixel".
(d) The script you have been given divides the 10-micron FITS image by the 12 micron FITS image, and then calculates the sum of all the pixels in the ratio image. The script works with test data that was supplied with the script, but when you try it with your data, all you get for the sum of the pixels is NaN. Can you suggest why this might be happening, and what you can do to fix the problem? *Hint: what is the purpose of the NaN designation in the C language?*

**Question 6 (7 marks)**

a) Explain in simple words, what is meant by a Fourier decomposition of a time-varying signal, y(t).

b) What physical quantity will the Fourier transform F, of the time varying signal y(x) be a function of? (i.e. what is the other member of the Fourier transform pair involving a quantity varying as a function of time?)

c) Sketch both the function and its Fourier transform for the following cases:

I. A delta function
II. A Gaussian function
III. A 2-dimensional circular aperture (your sketch should be 2-dimensional.