THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – NOVEMBER 2012

PHYS2060 – THERMAL PHYSICS
PHYS2210 – ELECTROMAGNETISM &THERMAL PHYSICS Paper 2

Time allowed – 2 hours

Total number of questions – 4

Attempt **ALL** questions

The questions are of EQUAL value

This paper may be retained by the candidate

Candidates may not bring their own calculators.

Answers must be written in ink. Except where they

are expressly required, pencils may only be used for

drawing, sketching or graphical work.

Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number $N_A = 6.022 \times 10^{23}$ mol^{-1}
Real gas constant $R = 8.314$ J/K.mol

Specific heat of liquid H_2O = 4.18 J/gK
Latent heat of the liquid-solid transition for H_2O = 333 J/g
Latent heat of the liquid-gas transition for H_2O = 2270 J/g
Adiabatic constant for N_2 g = 1.4
Specific heat at constant pressure for N_2 C_P = 29.12 J mol^{-1} K^{-1}
Specific heat at constant pressure for N_2 C_V = 20.8 J mol^{-1} K^{-1}
Molar mass of air = 29 g/mol

Partial derivatives

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \; ; \quad \left(\frac{\partial y}{\partial x}\right)_z = \frac{1}{\left(\frac{\partial x}{\partial y}\right)_z} \; ; \left(\frac{\partial y}{\partial x}\right)_z = -\frac{\left(\frac{\partial z}{\partial x}\right)_y}{\left(\frac{\partial z}{\partial y}\right)_x}$$

Thermodynamic quantities

$$\kappa_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T \quad \text{Isothermal compressibility} \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad \text{Expansivity}$$

$$\kappa_S = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_S \quad \text{Adiabatic compressibility} \qquad c_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V$$

$$c_P = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_P + \frac{P}{n}\left(\frac{\partial V}{\partial T}\right)_P$$

Thermodynamic potentials

$F=U-TS+\mu N$ $dF=-SdT-PdV+\mu dN$ Helmholtz Free Energy
$H=U+PV+\mu N$ $dH=TdS+VdP+\mu dN$ Enthalpy
$G=U-TS+PV+\mu N$ $dG=-SdT+VdP+\mu dN$ Gibbs Free Energy

Ideal gas

$PV=NkT$

$$U = \frac{1}{\gamma - 1}NkT$$

$PV^\gamma = const$ for Adiabatic process

Efficiency of Carnot cycle

$$\eta = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{T_c}{T_h}$$

QUESTION 1

The Otto cycle has four stages:
A→B – an isochoric depressurisation
B→C – an adiabatic compression
C→D – an isochoric pressurisation
D→A – an adiabatic expansion.

Assume that the working fluid for an ideal Otto engine is an ideal gas.

(i)   Sketch the ideal thermodynamic processes in the Otto cycle on a P-V diagram. On your diagram, indicate the direction of the path in the cycle. Mark processes where heat enters and leaves the cycle. Also mark processes where the cycle does external work and where work is done on the system.

(ii)   Derive an expression for the work done by one cycle of the Otto engine in terms of V_H, V_L, P_A, P_B, P_C, P_D and γ .

(iii)   Rewrite this expression for the work done by one cycle of the Otto engine in terms of T_A, T_B, T_C, T_D, n and γ .

(iv)   Derive an expression for the heat input into the Otto engine in terms of end point temperatures plus n and γ .

(v)   Derive an expression for the heat output from the Otto engine in terms of end point temperatures plus n and γ .

(vi)   Derive an expression for the efficiency of the Otto cycle in terms of the four endpoint temperatures T_A, T_B, T_C & T_D.

(vii)   Finally, show that the efficiency of the Otto cycle can be expressed as:

$$\eta = 1 - \frac{1}{r^{\gamma-1}}$$

where r is the compression ratio.

QUESTION 2

PART A

    (i)   Show that for an ideal gas, the specific heat at constant pressure is given by:

$$c_P = \frac{\gamma}{\gamma - 1} R$$

        where γ is the adiabatic constant and R is the universal gas constant.

PART B

The fundamental equation for a single component system can be expressed by the first order homogeneous equation:

$$U = U(S,V,N)$$

where first order homogeneous implies that for any λ:

$$\lambda U = U(\lambda S, \lambda V, \lambda N)$$

    (ii)   Using this property (or otherwise), show that for a single component system:

$$U = TS - PV + \mu N$$

    (iii)   Using this (or otherwise), derive the Gibbs-Duhem equation:

$$0 = SdT - VdP + nd\mu$$

    (iv)   Under conditions where temperature and pressure are constant, show that for a multicomponent system, the differential of the Gibbs Free energy (or Gibbs Function) is given by:

$$(dG)_{T,P} = \sum_j \mu_j dn_j$$

    (v)   Give an application to show the utility of this last equation.

QUESTION 3

PART A

(i) What is meant by a quasi-static process? Give an example.

(ii) Can a process be reversible and yet not approximated by a quasi-static process? If so, give an example, if not explain how a quasi-static process can always be constructed.

(iii) Can a process be quasi-static and yet not reversible? If so, give an example.

PART B

The Clausius inequality for a cyclic processes can be written as:

$$\oint \frac{dQ}{T} \leq 0$$

(iv) Explain the meaning of the left hand side of this equation for a reversible processes.

(v) Use this inequality to show that for a cyclic process with an irreversible step, the entropy change after one complete cycle is greater than zero.

PART C

The Third Law of Thermodynamics concerns the behaviour of substances at temperatures near absolute zero.

(vi) Explain what is meant by the Third Law, preferably with reference to one of the statements of the Third Law.

The Third Law of Thermodynamics has consequences for thermodynamic properties of a system as it approaches absolute zero. These properties include: thermal expansivity, the slope of phase transition curves and heat capacity.

(vii) Choose one of these properties and explain what happens to it as the temperature approaches absolute zero.

QUESTION 4

PART A

An ideal monatomic gas is contained in a piston whose state is characterised by the extensive parameters (U_0, S_0, V_0, N_0) and intensive parameters (T_0, P_0, μ_0). The system is placed in thermal contact with a heat reservoir at constant temperature T_0, hence, all processes are isothermal. The plunger in the piston moves slowly out of the piston so that the gas expands quasi-statically to a final volume which is twice the initial volume, $V_f = 2V_0$.
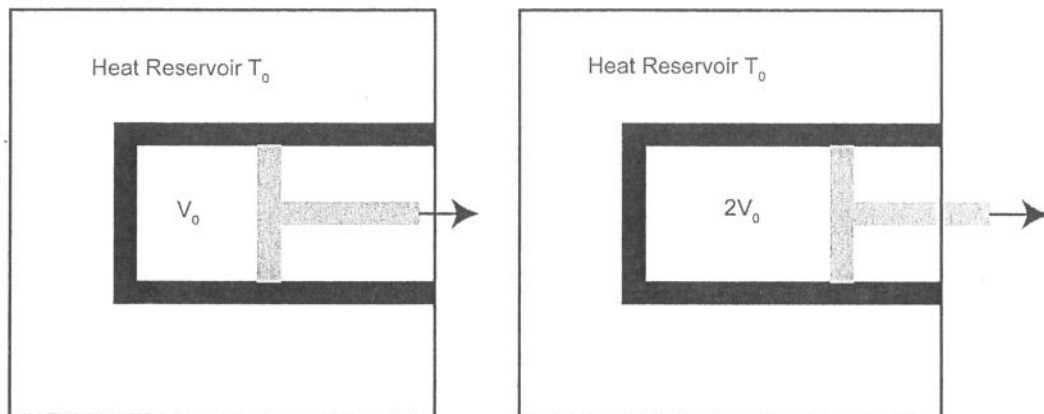

(i)   Calculate the work done by the gas as it expands from V_0 to $V_f = 2V_0$.

(ii)   Prove that the internal energy U does not change during this expansion.

(iii)   Calculate the amount of heat that enters the gas during this isothermal expansion.

(iv)   Calculate the change in entropy that occurs during the expansion.

(v)   What is the source of this entropy increase?

PART B

A new cylinder is wrapped in thermal insulation so that all processes are adiabatic. This cylinder has two chambers separated by a thin membrane. On the left side of the partition is an ideal monatomic gas that is in an identical state to the one described at the start of Part A. It is described by the same state parameter values: (U_0, S_0, V_0, N_0) and (T_0, P_0, μ_0). The chamber on the right side of the partition has the same volume V_0, but it is evacuated (empty). The partition separating the chambers is broken, so

that the gas expands irreversibly to fill the whole chamber of volume $V_f = 2V_0$.
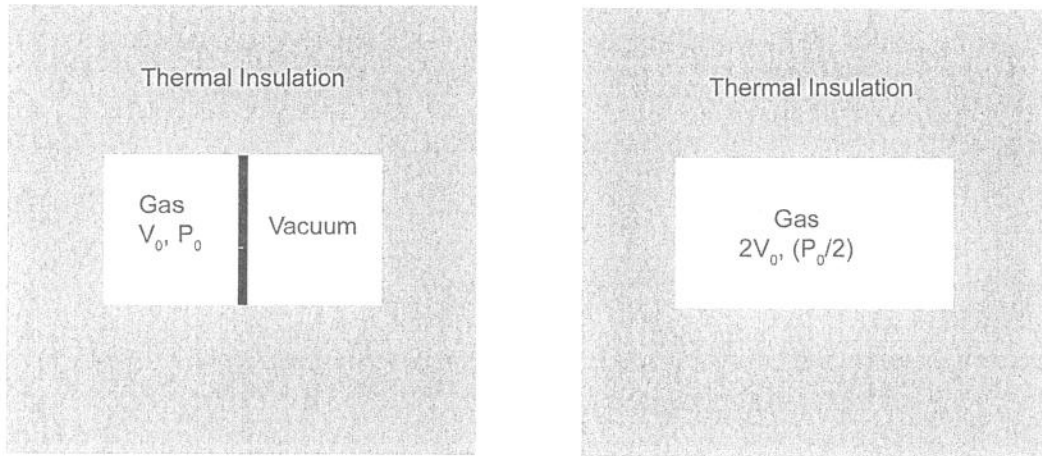
(vi) Determine the change of entropy that occurs during this expansion.

(vii) How does it compare with the quasi-static isothermal case in Part A.

(viii) What is the source of entropy increase?