**Final test 2002**
**Thermal Physics 2060**
Total number of questions **4**. Answer all the questions.

## Question 1

**a. (2 marks)** Sketch a typical graph of interatomic forces between two atoms in a real gas as a function of interatomic separation.

The van der Waals equation of state for one mole of a gas is of the form

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT.$$

**b. (2 marks)** Explain the significance of the term $b$. How could you relate it to a feature of your graph in question (**a**)?

**c. (2 marks)** What feature in your graph in question (**a**) is related to the term $\frac{a}{v^2}$? Give a simple argument to account for its sign and the exponent of $v$.

**d. (4 marks)** Using $P - v$ plot sketch a set of van der Waals isotherms. Clearly indicate regions corresponding to (i) gas, (ii) liquid and (iii) mixed state. At some isotherm indicate also parts corresponding to (iv) superheated liquid, (v) supercooled gas, and (vi) mechanically unstable state. Indicate also (vii) the critical point.

## Question 2

Consider a quantum particle moving in the oscillator potential. Energy levels of the particle are given by the formula

$$\epsilon_n = \hbar\omega n, \quad n = 0, 1, 2, 3...$$

Degeneracy of the each level is 1. The system is in the heat bath with temperature $T$.
**a. (6 marks)** Derive an expression for the probability to find the particle on the level n.
**b. (4 marks)** Calculate the average energy of the particle.
You may use without prove the sums of the following infinite progressions

$$1 + x + x^2 + x^3 + x^4... = \frac{1}{1-x},$$

$$x + 2x^2 + 3x^3 + 4x^4... = \frac{x}{(1-x)^2},$$

## Question 3

The 1st law of thermodynamics formulated for the internal energy $U$ reads

$$dU = TdS - PdV.$$

Other thermodynamic potentials are defined as $F = U - TS$ (Helmholtz free energy), $G = F + PV$ (Gibbs free energy), $H = U + PV$ (enthalpy).

**a. (3marks)** Starting from the 1st law formulated for the internal energy derive the following formulations of the 1st law in terms of $F$, $G$, and $H$:

$$dF = -SdT - PdV,$$

$$dG = -SdT + VdP,$$

$$dH = TdS + VdP.$$

**b. (4 marks)** Derive FOUR Maxwell relations for P, V, T and S.

**c. (3 marks)** Prove that

$$TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV.$$

## Question 4

The Helmholtz free energy of a gas is given by

$$F = -RT\ln(V/V_0) - AT\left[\ln(T/T_0) - 1\right] - \frac{B}{V},$$

where $R$, $A$, $B$, $V_0$, and $T_0$ are some constants

**a. (3 marks)** Derive equation of state of the gas.

**b. (3 marks)** Derive an expression for entropy of the gas.

**c. (2 marks)** Derive an expression for the internal energy $U$ of the gas.

**c. (2 marks)** Derive an expression for the heat capacity at constant volume $C_V$.