PHYS2210/2050 Electromagnetism T2 2015: Mid-session test

Monday September 7, 2015, 2-3pm

Question 1

Consider two point charges $+Q$ and $-Q$ placed a distance d apart. See figure 1 below.

(a) Compute the potential $V(r, \theta)$ at the point P.

(b) Show that in the limit $r \gg d$, the potential is well approximated by the dipole approximation

$$V_{\text{dipole}}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2},$$

where $\vec{p}$ is the electric dipole moment.

(c) Suppose instead of $+Q$ and $-Q$, both point charges are positive. Describe (without calculations) the monopole and dipole components of the resulting potential.

Question 2

The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. See figure 1 below. Each slab has thickness s, so that the total distance between the plates is $2s$. Slab 1has a dielectric constant of ϵ_1, and slab 2 has a dielectric constant of ϵ_2. The free charge density on the top plate is σ and on the bottom plate is $-\sigma$.

(a) Find the electric displacement in each slab.

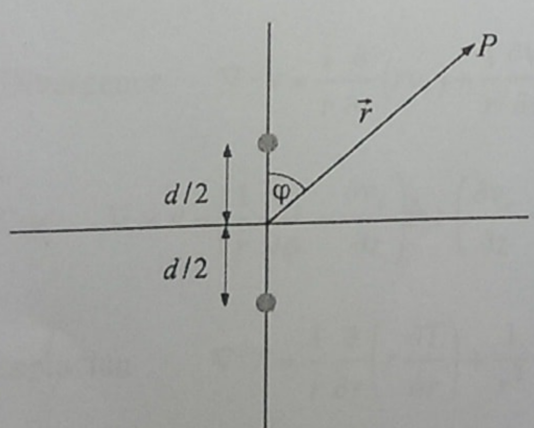
(b) Find the electric field in each slab.

(c) Find the polarisation in each slab.

(d) Find the potential difference between the plates.

(e) Find the location and amount of all bound charge.

(f) Now that you know all charges (free and bound), recalculate the field in each slab, and compare with your answers to (b).



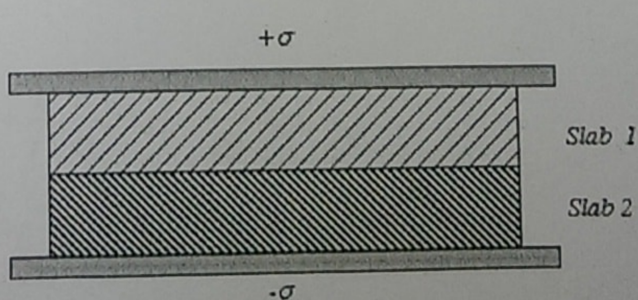


Figure 1: *Left*: Question 1. *Right*: Question 2.

CARTESIAN: $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\qquad d\tau = dx\,dy\,dz$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial x}\hat{\mathbf{x}} + \dfrac{\partial T}{\partial y}\hat{\mathbf{y}} + \dfrac{\partial T}{\partial z}\hat{\mathbf{z}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

Curl $\quad \nabla \times \mathbf{v} = \hat{\mathbf{x}}\left(\dfrac{\partial v_z}{\partial y} - \dfrac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\dfrac{\partial v_x}{\partial z} - \dfrac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)$

Laplacian $\quad \nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}$

SPHERICAL: $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\, d\phi\,\hat{\boldsymbol{\phi}}$; $\qquad d\tau = dl_r dl_\theta dl_\phi = r^2 \sin\theta\, dr d\theta d\phi$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 v_r\right) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \dfrac{1}{r\sin\theta}\dfrac{\partial v_\phi}{\partial \phi}$

Curl $\quad \nabla \times \mathbf{v} = \dfrac{1}{r\sin\theta}\left[\dfrac{\partial}{\partial \theta}(\sin\theta v_\phi) - \dfrac{\partial v_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \dfrac{1}{r}\left[\dfrac{1}{\sin\theta}\dfrac{\partial v_r}{\partial \phi} - \dfrac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \dfrac{1}{r}\left[\dfrac{\partial}{\partial r}(r v_\theta) - \dfrac{\partial v_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}$

Laplacian $\quad \nabla^2 T = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial T}{\partial r}\right) + \dfrac{1}{r^2 \sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta \dfrac{\partial T}{\partial \theta}\right) + \dfrac{1}{r^2 \sin^2\theta}\dfrac{\partial^2 T}{\partial \phi^2}$

CYLINDRICAL: $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\qquad d\tau = r dr\, d\phi\, dz$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}} + \dfrac{\partial T}{\partial z}\hat{\mathbf{z}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{1}{r}\dfrac{\partial}{\partial r}(r v_r) + \dfrac{1}{r}\dfrac{\partial v_\phi}{\partial \phi} + \dfrac{\partial v_z}{\partial z}$

Curl $\quad \nabla \times \mathbf{v} = \left(\dfrac{1}{r}\dfrac{\partial v_z}{\partial \phi} - \dfrac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\dfrac{\partial v_r}{\partial z} - \dfrac{\partial v_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(r v_\phi) - \dfrac{\partial v_r}{\partial \phi}\right)\hat{\mathbf{z}}$

Laplacian $\quad \nabla^2 T = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r \dfrac{\partial T}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 T}{\partial \phi^2} + \dfrac{\partial^2 T}{\partial z^2}$

TRIPLE PRODUCTS

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

PRODUCT RULES

(3) $\nabla(fg) = f\nabla g + g\nabla f$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

SECOND DERIVATIVES

(9) $\nabla\cdot(\nabla\times\mathbf{v}) = 0$

(10) $\nabla\times(\nabla T) = 0$

(11) $\nabla\times(\nabla\times\mathbf{v}) = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$

FUNDAMENTAL THEOREMS

Gradient Theorem $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T)\cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a})$

Divergence Theorem $\int_{volume} (\nabla\cdot\mathbf{v})d\tau = \oint_{surface} \mathbf{v}\cdot d\mathbf{a}$

Curl Theorem $\int_{surface} (\nabla\times\mathbf{v})\cdot d\mathbf{a} = \oint_{boundary\ line} \mathbf{v}\cdot d\mathbf{l}$