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THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

Session 2, 2012

PHYS2050

Electromagnetism

PHYS2210

Electromagnetism and Thermal Physics

1. Time Allowed: 2 hours
2. Total number of questions: 5
3. Marks available for each question are shown in the examination paper. The total number of marks is 50.
4. Attempt ALL questions!
5. Answer questions 1-2 in one answer book, and the remaining questions (3-5) in a separate answer book
6. University-approved calculators may be used.
7. Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work. Do not use red ink.
8. The exam paper may be retained by the candidate.

PHYS2050 — Definitions and Formulae

Gradient	$\nabla f = \frac{\partial f}{\partial x}\,\hat{\imath} + \frac{\partial f}{\partial y}\,\hat{\jmath} + \frac{\partial f}{\partial z}\,\hat{\mathbf{k}}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
Curl	$\nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{\imath} + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{\jmath} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{\mathbf{k}}$
Laplacian	$\nabla \cdot (\nabla f) = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
Identities	$\nabla \times (\nabla f) = 0 \qquad \nabla.(\nabla \times \mathbf{A}) = 0$ $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$
Volume element	$dx\,dy\,dz$ (Cartesian coordinates) $r\,dr\,d\phi\,dz$ (Cylindrical coordinates) $r^2\,\sin\theta\,dr\,d\theta\,d\phi$ (Spherical coordinates)
Gradient theorem	$\int_a^b (\nabla f) \cdot \mathbf{d\ell} = f(b) - f(a)$
Divergence Theorem	$\int_S \mathbf{A} \cdot \mathbf{ds} = \int_V (\nabla \cdot \mathbf{A})\ \mathrm{dv}$
Stokes' Theorem	$\int_L \mathbf{A} \cdot \mathbf{d\ell} = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{ds}$
Coulomb's Law	$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{r}}$
Electric Field	$\mathbf{F} = Q\,\mathbf{E} \qquad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v}{r^2}\,\hat{\mathbf{r}}\ \mathrm{dv}$
Gauss' Law	$\int_S \mathbf{E} \cdot \mathbf{ds} = \frac{Q}{\epsilon_0} \qquad \nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon_0}$
Electric Potential	$V(b) - V(a) = -\int_a^b \mathbf{E} \cdot \mathbf{d\ell} \qquad \mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r})$ $\nabla^2 = -\frac{\rho_v}{\epsilon_0} \qquad V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v}{r}\ \mathrm{dv}$
Current Density	$\mathbf{J} = \rho_v \mathbf{v} = \rho_v \mu\,\mathbf{E} = \sigma \mathbf{E}$
Charge Conservation	$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$
Stored Energy	$W = \frac{1}{2}\sum q_i V(\mathbf{r}_i)$ $W = \frac{1}{2}\int_V V(\mathbf{r})\rho(\mathbf{r})\ \mathrm{dv} = \frac{1}{2}\int_V \epsilon E^2\ \mathrm{dv} = \frac{1}{2}\int_V \mathbf{D} \cdot \mathbf{E}\ \mathrm{dv}$
Electric Diplacement	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad \int_S \mathbf{D} \cdot \mathbf{ds} = Q_f$
Linear Media	$\mathbf{P} = \chi_e \epsilon_0 \mathbf{E} \qquad \mathbf{D} = \epsilon_0(1 + \chi_e)\mathbf{E} = \epsilon \mathbf{E}$
Capacitance	$C = \frac{Q}{V}$

DC Circuits:

Ohm's Law: $\Delta V = IR$ resistance, $R = \rho l/A$ [Ω]

Kirchhoff's Laws: (1) $\Sigma I = 0$ at a junction
(2) $\Sigma \mathcal{E} - \Sigma IR = 0$ around each loop

Joule heating: power dissipated, $P = I\Delta V = I^2 R = (\Delta V)^2/R$ [W]

Ohm's law: $\mathbf{J} = \sigma \mathbf{E}$ power dissipated/unit volume $= \mathbf{J}\cdot\mathbf{E} = \sigma E^2$

Current density in a metal: $\mathbf{J} = -nev_d$

Magnetism:

Magnetic force on a moving charge q is $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$

Magnetic force on a current element $I\mathbf{d\ell}$ is $\mathbf{dF} = I\mathbf{d\ell}\times\mathbf{B}$

There are no 'magnetic charges', so for a closed surface S

$$\int_S \mathbf{B}\cdot\mathbf{dS} = 0 \text{ or } \int_V \nabla\cdot\mathbf{B}\, dV = 0 \qquad \text{so: } \nabla\cdot\mathbf{B} = 0$$

Biot-Savart Law:

$\mathbf{B}$ field from a moving charge q', with velocity $\mathbf{v}'$: $\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q'}{r^2}(\mathbf{v}'\times\hat{\mathbf{r}})$

element of magnetic field produced by a current element $I\mathbf{d\ell}$ is:

$$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{I\mathbf{d\ell}\times\hat{\mathbf{r}}}{r^2} \qquad \text{or} \qquad |dB| = \frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2}$$

Force between two current elements $= \frac{\mu_0}{4\pi}\frac{II'\,\mathbf{d\ell}'\times(\mathbf{d\ell}\times\hat{\mathbf{r}})}{r^2}$

(like currents attract, unlike currents repel)

so: force/unit length between two long parallel current-carrying wires is

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{II'}{r} \qquad [\mathrm{Nm^{-1}}]$$

Force on wire of length ℓ, perpendicular to magnetic field: $F = BI\ell$ [N]

Particle of mass m, charge q moving perpendicular to magnetic field:
cyclotron radius, $r = mv/(qB)$ [m]; cyclotron frequency, $f = qB/(2\pi m)$ [Hz]

Hall efect: Hall coefficient, $R_H = 1/nq$, charge mobility $v_d/E = \sigma R_H$.

Ampere's law: $\oint \mathbf{B}\cdot\mathbf{d\ell} = \mu_0 I$ (I is current linked by the closed path)

Differential form: $\nabla\times\mathbf{B} = \mu_0\mathbf{J}$

Magnetic field at distance r from a long straight wire: $\mathbf{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$
($\mathbf{B}$ is in circles around wire)

Magnetic field on axis of a circular wire loop of radius R carrying current I is:

$$B_z = \frac{\mu_0}{2\pi}\frac{I\,\pi R^2}{(z^2+R^2)^{\frac{3}{2}}}$$

Magnetic dipole moment of a small circular current loop is $m = I\pi R^2$
general formula is $m = IA$ [$\mathrm{Am^2}$]

Axial magnetic field inside a long solenoid is $B = \mu_r \mu_0 n I$, where n is the number of turns per unit length

Faraday's Law for EMF by induction: $\oint \mathbf{E} \cdot \mathbf{dl} = \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \mathbf{B} \cdot \mathbf{dS}$

Or: EMF = rate of cutting magnetic flux.

Differential form: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

Mutual Inductance: $L_{12} = \frac{\text{Flux in circuit 2}}{\text{Current in coil 1}} = \frac{\Phi_2}{I_1} \quad (= L_{21})$

Self Inductance: $L = \frac{\Phi}{I}$ $\qquad V = -L\frac{dI}{dt}$ $\qquad$ Magnetic energy: $U = \frac{1}{2}LI^2$

Self Inductance of a solenoid: $L = \mu_r \mu_0 \frac{N^2}{\ell} A$

Energy density in magnetic field: $u = \frac{1}{2}\frac{B^2}{\mu_r \mu_0} = \frac{1}{2}\mathbf{B} \cdot \mathbf{H}$

Magnetic media:

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi_m)\mathbf{H} = \mu_r \mu_0 \mathbf{H}$ $\qquad$ *ie* $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$

$\nabla \cdot \mathbf{B} = 0$, so $\nabla \cdot \mathbf{H} + \nabla \cdot \mathbf{M} = 0$

Ampère's law becomes: $\nabla \times \mathbf{H} = \mathbf{J}_{free}$

At a boundary, $B'_\perp = B_\perp$ and $H'_\parallel = H_\parallel$

Maxwell's Equations

In a vacuum:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Lorentz force law: $\quad \mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

EM Waves:

Wave equation for $\mathbf{E}$ in free space: $\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$ $\qquad$ *ie* $c = 1/\sqrt{\mu_0 \epsilon_0}$

(in a medium: $v = 1/\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0} = c/n$, $\quad n$ = refractive index)

Solution: $E_x = E_0 \sin(kx - \omega t)$ for monochromatic wave travelling in +ve x-direction.

$\mathbf{E}$, $\mathbf{B}$ and the direction of propagation $\hat{\mathbf{k}}$ are mutually perpendicular:

$\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{k}} \cdot \mathbf{E} = 0 \qquad \hat{\mathbf{k}} \cdot \mathbf{B} = 0 \qquad c\mathbf{B} = \hat{\mathbf{k}} \times \mathbf{E}$

The direction of $\mathbf{E}$ is the direction of polarization of the E-M wave.

Poynting vector: $\mathbf{N} = \mathbf{E}\times\mathbf{H} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{E^2}{\mu_0 c}$ $\quad$ [W.m^{-2}]

Useful information

$\epsilon_0 = 8.854 \text{ x } 10^{-12}$ Fm^{-1}
$(1/4\pi\epsilon_0) = 8.99 \text{ x } 10^9$ mF^{-1}
$\mu_0 = 4\pi \text{ x } 10^{-7}$ Hm^{-1}
speed of light, $c = 3 \times 10^8$ ms^{-1}
elementary charge, e = 1.60 x 10^{-19} C
1eV = 1.60 x 10^{-19} J
electron mass = 9.11 x 10^{-31} kg
proton mass = 1.67 x 10^{-27} kg
Avogadro's number, $N_A = 6.022 \text{ x } 10^{23}$ mol^{-1}
Boltzmann's const, $k_B = 1.381 \text{ x } 10^{-23}$ JK^{-1}

Question 1 (10 marks)

Two long, coaxial, cylindrical metal tubes of length l stand vertically in a tank of oil (dielectric constant ε). The inner radius is a and the outer radius is b, and they have charges +Q and -Q, respectively. The stray field at the boundary can be neglected.

(a) Calculate the electric potential between the two uniformly charged tubes when there is no oil present.

(b) When the capacitor is placed in the oil, the dielectric is pulled into the space between the tubes. Calculate the force pulling the oil into the capacitor when it is at height h.

(c) Calculate the gravitational force pulling the oil down, and hence find an expression that allows one to determine the final height h that the oil rises to. Do not actually solve for h it gets pretty messy.

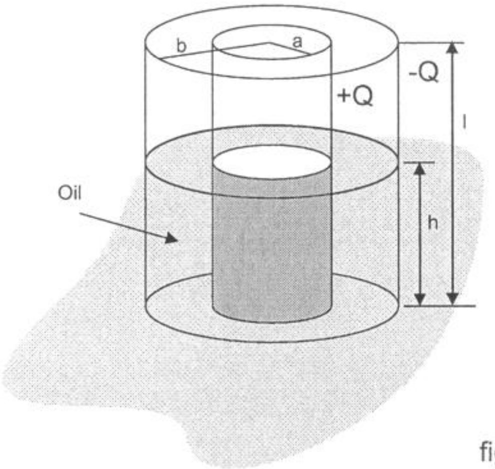

fig. 1

Question 2 (10 marks)

(a) For the circuit shown in diagram 2a, find the currents in each of the two batteries.

(b) The 5 V battery is now removed, and replaced by a wire of negligible resistance (a short-circuit: see diagram 2b.) Find the new current in the 10 V battery.

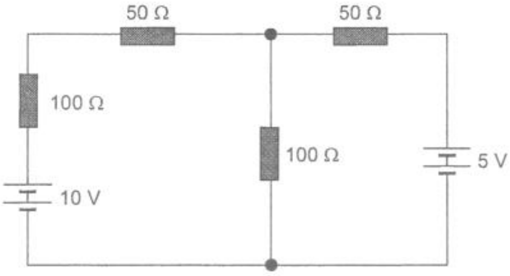

fig. 2a

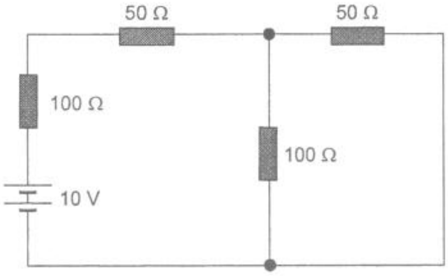

fig. 2b

Question 3 (10 marks)

A long solenoid of circular cross-section contains n turns per m, and carries a current of I_0 A. A small circular coil of radius r m and N turns, of resistance $R\ \Omega$ is placed inside the first coil, with its axis initially parallel to the axis of the first coil. The second coil is free to rotate about a line which passes through a diameter of both coils. (See diagram 3.) Find:

(a) The initial flux in the second coil, before it starts to rotate.

(b) The flux in the second coil, when it has rotated through θ radians.

(c) The current in the second coil at this angle, if it is rotating at an angular velocity of ω radians per second.

(d) The total charge which has flowed in the second coil, when it has rotated through π radians.

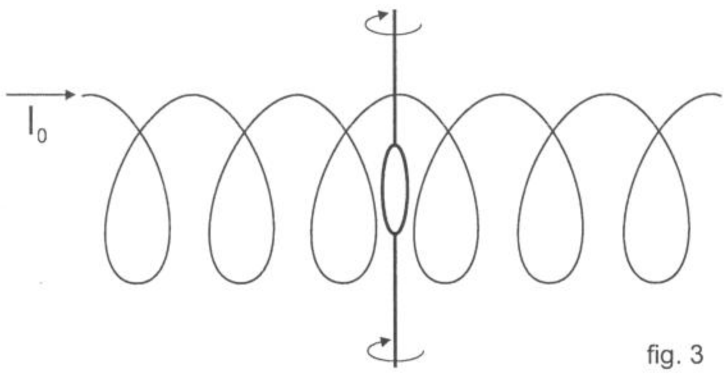

fig. 3

Question 4 (10 marks)

A satellite is in a geo-stationary orbit (35,786 km above the equator, but let's suppose it is 40,000 km from Sydney.) It broadcasts a TV signal which is not isotropic, but the intensity in the direction of Sydney is equivalent to a uniform intensity of 20 kW over a sphere.

What is the rms electric field received at Sydney ?
(You can assume a plane wave at this distance.)

Question 5 (10 marks)

Starting from Maxwell's equations for the **E** and **B** fields in a vacuum with no free charges or currents, show that the **E** field obeys the wave equation, and show that the speed of electromagnetic waves in a vacuum is given by $c = 1/\sqrt{\mu_0 \epsilon_0}$.

END OF EXAM