THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

PHYS2050 – Electromagnetism

Session 2, 2011

1. Time allowed – 2 hours
2. Total number of questions – 5
3. Total marks available – 50
4. Marks available for each question are shown
5. Answer ALL questions
6. Answer questions 1 and 2 in one answer book, and the remaining questions (3-5) in a separate answer book
7. University-approved calculators may be used
8. All answers must be written in ink. Except where they are explicitly required, pencils may only be used for drawing, sketching or graphical work. Do not use red ink
9. This paper may be retained by the candidate.

PHYS2050 **Definitions and Formulae**

Gradient	$\nabla f = \frac{\partial f}{\partial x}\,\hat{\mathbf{i}} + \frac{\partial f}{\partial y}\,\hat{\mathbf{j}} + \frac{\partial f}{\partial z}\,\hat{\mathbf{k}}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
Curl	$\nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{\mathbf{i}} + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{\mathbf{j}} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{\mathbf{k}}$
Laplacian	$\nabla \cdot (\nabla f) = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$
Identities	$\nabla \times (\nabla f) = 0 \qquad \nabla.(\nabla \times \mathbf{A}) = 0$ $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$
Volume element	$dx\,dy\,dz$ (Cartesian coordinates) $\quad r\,dr\,d\phi\,dz$ (Cylindrical coordinates) $r^2\,\sin\theta\,dr\,d\theta\,d\phi$ (Spherical coordinates)
Gradient theorem	$\int_a^b (\nabla f) \cdot \mathbf{d}\boldsymbol{\ell} = f(b) - f(a)$
Divergence Theorem	$\int_S \mathbf{A} \cdot \mathbf{ds} = \int_V (\nabla \cdot \mathbf{A})\,\mathrm{dv}$
Stokes' Theorem	$\int_L \mathbf{A} \cdot \mathbf{d}\boldsymbol{\ell} = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{ds}$
Coulomb's Law	$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{r}}$
Electric Field	$\mathbf{F} = Q\,\mathbf{E} \qquad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v}{r^2}\,\hat{\mathbf{r}}\,\mathrm{dv}$
Gauss' Law	$\int_S \mathbf{E} \cdot \mathbf{ds} = \frac{Q}{\epsilon_0} \qquad \nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon_0}$
Electric Potential	$V(b) - V(a) = -\int_a^b \mathbf{E} \cdot \mathbf{d}\boldsymbol{\ell} \qquad \mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r})$ $\nabla^2 = -\frac{\rho_v}{\epsilon_0} \qquad V = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v}{r}\,\mathrm{dv}$
Current Density	$\mathbf{J} = \rho_v \mathbf{v} = \rho_v \mu\,\mathbf{E} = \sigma \mathbf{E}$
Charge Conservation	$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$
Stored Energy	$W = \frac{1}{2}\sum q_i V(\mathbf{r}_i)$ $W = \frac{1}{2}\int_V V(\mathbf{r})\rho(\mathbf{r})\,\mathrm{dv} = \frac{1}{2}\int_V \epsilon E^2\,\mathrm{dv} = \frac{1}{2}\int_V \mathbf{D} \cdot \mathbf{E}\,\mathrm{dv}$
Electric Diplacement	$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \qquad \int_S \mathbf{D} \cdot \mathbf{ds} = Q_f$
Linear Media	$\mathbf{P} = \chi_e \epsilon_0 \mathbf{E} \qquad \mathbf{D} = \epsilon_0(1 + \chi_e)\mathbf{E} = \epsilon \mathbf{E}$
Capacitance	$C = \frac{Q}{V}$

Magnetism:

Magnetic force on a moving charge q is $\quad \mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

Magnetic force on a current element $I\mathbf{d}\boldsymbol{\ell}$ is $\quad \mathbf{dF} = I\mathbf{d}\boldsymbol{\ell} \times \mathbf{B}$

There are no 'magnetic charges', so for a closed surface S

$$\int_S \mathbf{B}\cdot\mathbf{dS} = 0 \;\text{ or }\; \int_V \nabla\cdot\mathbf{B}\, dV = 0 \qquad \text{so: } \nabla\cdot\mathbf{B} = 0$$

Biot-Savart Law:

$\mathbf{B}$ field from a moving charge q', with velocity $\mathbf{v}'$: $\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q'}{r^2}(\mathbf{v}' \times \hat{\mathbf{r}})$

element of magnetic field produced by a current element $I\mathbf{d}\boldsymbol{\ell}$ is:

$$\mathbf{dB} = \frac{\mu_0}{4\pi}\frac{I\mathbf{d}\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2} \qquad \text{or} \qquad |dB| = \frac{\mu_0}{4\pi}\frac{I\, d\ell \sin\theta}{r^2}$$

Force between two current elements $= \frac{\mu_0}{4\pi}\frac{II'\,\mathbf{d}\boldsymbol{\ell}' \times (\mathbf{d}\boldsymbol{\ell} \times \hat{\mathbf{r}})}{r^2}$

(like currents attract, unlike currents repel)

so: force/unit length between two long parallel current-carrying wires is

$$\frac{F}{\ell} = \frac{\mu_0}{2\pi}\frac{II'}{r} \qquad [\mathrm{Nm}^{-1}]$$

Force on wire of length ℓ, perpendicular to magnetic field: $F = BI\ell$ [N]

Particle of mass m, charge q moving perpendicular to magnetic field:
cyclotron radius, $r = mv/(qB)$ [m]; cyclotron frequency, $f = qB/(2\pi m)$ [Hz]

Hall efect: Hall coefficient, $R_H = 1/nq$, charge mobility $v_d/E = \sigma R_H$.

Ampere's law: $\oint \mathbf{B}\cdot\mathbf{d}\boldsymbol{\ell} = \mu_0 I$ (I is current linked by the closed path)

Differential form: $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Magnetic field at distance r from a long straight wire: $\mathbf{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$

($\mathbf{B}$ is in circles around wire)

Magnetic field on axis of a circular wire loop of radius R carrying current I is:

$$B_z = \frac{\mu_0}{2\pi}\frac{I\,\pi R^2}{(z^2 + R^2)^{\frac{3}{2}}}$$

Magnetic dipole moment of a small circular current loop is $m = I\pi R^2$
general formula is $m = IA$ [Am^2]

Axial magnetic field inside a long solenoid is $B = \mu_r\mu_0 nI$, where n is the number of turns per unit length

Faraday's Law for EMF by induction: $\oint \mathbf{E}\cdot\mathbf{dl} = \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \mathbf{B}\cdot\mathbf{dS}$

Or: EMF = rate of cutting magnetic flux.

Differential form: $\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$

Mutual Inductance: $L_{12} = \dfrac{\text{Flux in circuit 2}}{\text{Current in coil 1}} = \dfrac{\Phi_2}{I_1} \quad (= L_{21})$

Self Inductance: $L = \dfrac{\Phi}{I}$ $\qquad V = -L\dfrac{dI}{dt}$ $\qquad$ Magnetic energy: $U = \frac{1}{2}LI^2$

Self Inductance of a solenoid: $L = \mu_r\mu_0 \dfrac{N^2}{\ell} A$

Energy density in magnetic field: $u = \frac{1}{2}\dfrac{B^2}{\mu_r\mu_0} = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}$

Magnetic media:

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi_m)\mathbf{H} = \mu_r\mu_0\mathbf{H}$ $\qquad$ *ie* $\mathbf{H} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{M}$

$\nabla\cdot\mathbf{B} = 0$, so $\nabla\cdot\mathbf{H} + \nabla\cdot\mathbf{M} = 0$

Ampère's law becomes: $\nabla\times\mathbf{H} = \mathbf{J}_{free}$

At a boundary, $B'_\perp = B_\perp$ and $H'_\parallel = H_\parallel$

Maxwell's Equations

In a vacuum:

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

Lorentz force law: $\qquad \mathbf{F} = q\,(\mathbf{E}+\mathbf{v}\times\mathbf{B})$

Maxwell's equations in general media:

$$\nabla\cdot\mathbf{D} = \rho \qquad \nabla\times\mathbf{E} = -\dot{\mathbf{B}}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$$

EM Waves:

Wave equation for $\mathbf{E}$ in free space: $\nabla^2\mathbf{E} = \mu_0\epsilon_0\dfrac{\partial^2\mathbf{E}}{\partial t^2}$ $\qquad$ *ie* $c = 1/\sqrt{\mu_0\epsilon_0}$

(in a medium: $v = 1/\sqrt{\mu_r\mu_0\epsilon_r\epsilon_0} = c/n$, $\quad n$ = refractive index)

Solution: $E_x = E_0\sin(kx-\omega t)$ for monochromatic wave travelling in +ve x-direction.

$\mathbf{E}$, $\mathbf{B}$ and the direction of propagation $\hat{\mathbf{k}}$ are mutually perpendicular:

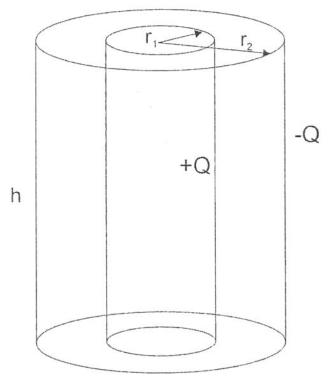
$\hat{\mathbf{E}}\times\hat{\mathbf{B}} = \hat{\mathbf{k}}$ $\qquad \hat{\mathbf{k}}\cdot\mathbf{E} = 0$ $\qquad \hat{\mathbf{k}}\cdot\mathbf{B} = 0$ $\qquad c\mathbf{B} = \hat{\mathbf{k}}\times\mathbf{E}$

The direction of $\mathbf{E}$ is the direction of polarization of the E-M wave.

Poynting vector: $\mathbf{S} = \mathbf{E}\times\mathbf{H} = \dfrac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})$ $\qquad$ [W.m^{-2}]

Question 1 (10 marks)

Two cylindrical uniformly charged plates



The inner radius is r_1 and the outer radius is r_2. Both plates are charged with $+Q$ and $-Q$, respectively. The height of both cylinders is h. The stray field at the boundary of the cylinder can be neglected.

(i) Calculate the electric field between the two cylindrical uniformly charged plates.

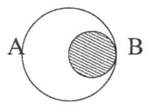
(ii) Calculate the electric potential between the two cylindrical uniformly charged plates.

(iii) Calculate the capacity of this cylindrical plate capacitor.

(iv) Calculated the energy of the electric field of the cylindrical plate capacitor.

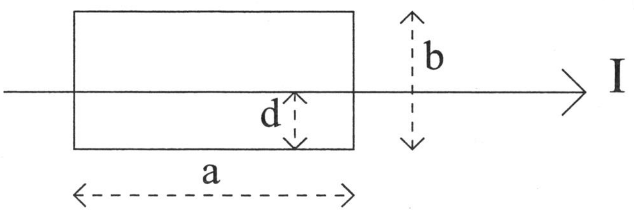
P.T.O.

Question 2 (10 marks)



A long cylindrical conductor of radius R and uniform resistivity has a cylindrical hole of half this radius, as shown in the cross-sectional digram above. It carries a steady current, I. Using the principle of superposition, find the ratio of the magnetic fields at the two ends of the diameter, A and B.
Hint: Using Ampère's law, consider the effect of a complete cylinder minus a cylinder carrying an opposite current. *Make sure that you use two currents of appropriate magnitudes.*

Question 3 (10 marks)



Find the mutual inductance between an infinitely long straight wire and a one-turn rectangular coil whose plane passes through the wire and two of whose sides are parallel to the wire. The sides parallel to the wire are of length a, the other sides are of length b and the side nearest the wire is a distance d from it (see figure above.)
Hint: The flux below the wire is cancelled by part of the flux above: what limits of integration should you use to find the *net* flux ?

P.T.O.

Question 4 (10 marks)

A coil with n turns per metre is wound on a long empty cylinder of length ℓ. A rod of magnetic material with relative permeability μ_r, of circular cross-section, which just fits inside the coil, is inserted to a distance x. Find the energy in the magnetic field, and hence the force per unit cross-sectional area on the rod, if the current I is maintained at a constant value.
Hint: consider this as two solenoids in series, one of length x, containing the rod, the other of length $(\ell - x)$, containing just air.

Question 5 (10 marks)

A radio station has an effective power of 20 kW at a frequency of 1 MHz. ('Effective' means this is the power it would need if it radiated uniformly over a sphere.)
Calculate the rms **E** and **B** field strengths at a point 50 km away from the transmitter. (You can assume a plane wave front at this distance.)

END OF EXAM

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PHYS2050 – Electromagnetism

Additional Information

$\epsilon_0 = 8.854 \text{ x } 10^{-12}$ Fm^{-1}
$(1/4\pi\epsilon_0) = 8.99 \text{ x } 10^9$ mF^{-1}
$\mu_0 = 4\pi \text{ x } 10^{-7}$ Hm^{-1}
speed of light, $c = 3 \times 10^8$ ms^{-1}
elementary charge, e = $1.60 \text{ x } 10^{-19}$ C
1eV = $1.60 \text{ x } 10^{-19}$ J
electron mass = $9.11 \text{ x } 10^{-31}$ kg
proton mass = $1.67 \text{ x } 10^{-27}$ kg
Avogadro's number, $N_A = 6.022 \text{ x } 10^{23}$ mol^{-1}
Boltzmann's const, $k_B = 1.381 \text{ x } 10^{-23}$ JK^{-1}