THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

FINAL EXAMINATION

JUNE 2015

PHYS2110 Quantum Physics and Laboratory (Quantum Paper)

PHYS2040 Quantum Physics

Time Allowed – 2 hours

Total number of questions – 5

Use separate booklets for Questions 1-2 inc. and for Questions 3-5 inc.

Total marks: 70 – Questions are not all of equal value

This paper may be retained by the candidate.

Students must provide their own UNSW approved calculators.

Answers must be written in ink. Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

The following information may be useful

Planck's constant $h = 6.626 \times 10^{-34}$ Js

Fundamental charge unit $e = 1.60 \times 10^{-19}$ C

Speed of light (vacuum) $c = 3.0 \times 10^{8}$ m/s

Electron mass = 9.1×10^{-31} kg

Neutron mass = 1.675×10^{-27} kg

Proton mass = 1.672×10^{-27} kg

Boltzmann's constant $k = 1.38 \times 10^{-23}$ JK^{-1}

Angstrom (Å) = 1.0×10^{-10} m

Permittivity constant $\varepsilon_o = 8.85 \times 10^{-12}$ Fm^{-1}

Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2

Time-independent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V\psi(x) = E\psi(x)$

Time-dependent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$

$$\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$$

$$\int x\sin^2(x)dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$$

$$\int x^2\sin^2(bx)dx = \frac{x^3}{6} - \left(\frac{x^2}{4b} - \frac{1}{8b^3}\right)\sin(2bx) - \frac{x\cos(2bx)}{4b^2}$$

$$\int_{-\infty}^{\infty} e^{-bx^2}dx = \sqrt{\frac{\pi}{b}}$$

$$\int_{0}^{\infty} x^n e^{-bx}dx = \frac{n!}{b^{n+1}}$$

$\sin(2\theta) = 2\sin\theta\cos\theta$

Bragg's law: $n\lambda = 2d\sin\theta$

Compton Shift: $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$

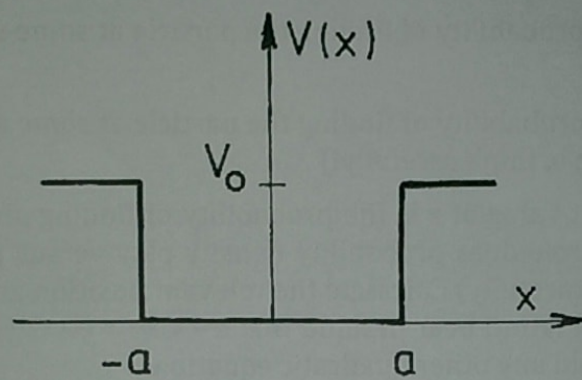
Question 1 (Marks 12)

Give a concise account of 2 **only** of the following 4 topics. Use diagrams and sketch graphs where appropriate to illustrate your account.

(i) The Ultraviolet catastrophe
(ii) The Compton effect
(iii) The Heisenberg Uncertainty Principle
(iv) The Stern-Gerlach experiment

Question 2 (Marks 18)

A particle is incident on a 1D potential as shown:



(a) Write down the form of the time-independent wavefunction $\psi(x)$ in the three regions $x < -a$, $-a < x < a$, and $x > a$, for the case where $E < V_0$. In each case, very briefly explain why the wavefunction takes this form (1 sentence will suffice).

(b) What are the wavefunction matching conditions required to hold at $x = \pm a$? Is it possible to obtain all four normalization prefactors using this set of matching conditions? Briefly explain why.

(c) Draw a schematic energy level diagram, assuming there are exactly three bound states. Sketch the wavefunction for each of these bound states. Comment briefly on any special features in any of these diagrams, particularly where quantum and classical predictions disagree.

(d) Discuss how the following will change when the depth of the well is increased.

(i) The ground state energy

(ii) The number of bound states

(iii) The probability of finding the particle in the classically forbidden region

(e) Suppose particles of energy $E > V_0$ are incident on this potential from the negative x direction. Are there any energies where the transmission coefficient $T \cong 1$ (for transmission from the region where $x < -a$ to the region where $x > a$)? If so, what energies and why?

Question 3 (Marks 9)

A particle with zero energy has a wavefunction: $\psi(x) = Ax\exp(-x^2/L^2)$

(a) Briefly explain how you would obtain the potential energy U as a function of x.

(b) Follow your method in (a) to obtain an expression for $U(x)$.

(c) Make a sketch of U vs x. Be sure to annotate all relevant points in your sketch (e.g., minima/maxima, intercepts with x-axis, etc.)

Question 4 (Marks 19)

An electron is described by the 1D wavefunction:

$$\psi(x) = 0 \qquad \text{for } x < 0$$

$$\psi(x) = (12a)^{1/2}(e^{-ax} - e^{-2ax}) \qquad \text{for } x > 0$$

where a is a real constant with dimensions of m^{-1}

(a) What is the probability of finding the particle at some specific position x for $x < 0$? Why?

(b) What is the probability of finding the particle at some specific position x for $x > 0$? Why? (n.b. think carefully!)

(c) Around what value of x is the probability of finding an electron the highest? Why? What role does probability density play versus probability here? (n.b. again, think carefully) Calculate the relevant position and give your answer in terms of $1/a$. (Hint: bear in mind that $e^{ax} \times e^{bx} = e^{(a+b)x}$ and treat the problem like you would any other quadratic equation.)

(d) Calculate the expectation value $\langle x \rangle$ of measurements of position performed on an ensemble of identically prepared electrons.

(e) Explain why the results of parts (c) and (d) are different.

(f) Is the wavefunction for the electron an eigenfunction of momentum? Give reasons for your answer.

Question 5 (Marks 12)

(a) The wavefunction of the ground state of the hydrogen atom is given by:

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0{}^3}} e^{-r/a_0}$$

where $a_0 = 4\pi\varepsilon_0\hbar^2/me^2$ is the Bohr radius

(i) Obtain an expression for the radial probability density $P(r)$, where $P(r)dr$ is the probability of finding the electron in a thin spherical shell between r and $r + dr$. Briefly explain any geometrical factors that might arise in your expression.

(ii) Sketch the form of $P(r)$. Make sure you calculate and annotate on your sketch the position of all zeroes, maxima and minima.

(b) The wavefunction can be more generally written as $\psi_{nlm}(r,\theta,\phi)$. Explain the physical meaning of the quantum numbers n, l, and m and state which values they can take.

(c) Consider the hydrogen atom in the $l = 3$ state. Calculate the magnitude of $\boldsymbol{L}$ and the allowed values of L_z. What is the corresponding set of angles that $\boldsymbol{L}$ can make with the z-axis?