THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – JUNE 2015

PHYS2010 – MECHANICS

PHYS2120 – MECHANICS & COMPUTATIONAL PHYSICS (Paper 1)

Time allowed – 2 hours

Total number of questions – 4

Answer ALL FOUR questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{c}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 e^{i\omega t}$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$
$$\theta = \pm\int \frac{\left(L/r^2\right)dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$

Lagrangian

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Question 1

For the damped harmonic oscillator, the equation of motion can be written as:

$$m\ddot{x}+c\dot{x}+kx=0$$

(a) explain all terms in this equation.

This results in three distinct types of solutions that represent three types of physical behaviour: overdamping, critical damping and underdamping.

(b) Explain each of these modes of physical behaviour. Use sketches to illustrate the motion of the particle in each case.

The above oscillator is driven by a harmonic driving force given by:

$$F(t)=F_o e^{i\omega t}$$

On solving the equations of motion for this driven damped harmonic oscillator, one finds that the amplitude, $A(\omega)$, as a function of frequency is given by:

$$A(\omega)=\frac{F_o/m}{\left[\left(\omega_o^2-\omega^2\right)^2+4\gamma^2\omega^2\right]^{\frac{1}{2}}}$$

(c) Explain the meaning of all the symbols in the equation for $A(\omega)$.
(d) Derive an equation for the resonant frequency of the system.
(e) Sketch $A(\omega)$ as a function of ω for various values of γ. Be sure to plot and label curves where the behaviour of the system changes.

The phase shift in the driven damped harmonic oscillator is given by:

$$\tan(\phi(\omega))=\frac{2\gamma\omega}{\omega_o^2-\omega^2}$$

(f) Explain the meaning of this equation.
(g) Plot ϕ as a function of ω for several values of γ. Label all important parameters on your graph (common points, asymptotes, inflection points etc).

Question 2

An isotropic central force is given by:

$$\mathbf{F} = F(r)\hat{\mathbf{r}}$$

The natural coordinates for analysing central forces are plane polar coordinates (r,θ). In these coordinates, the acceleration is given by:

$$\ddot{\mathbf{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{\theta}$$

where $\hat{\mathbf{r}}$ and $\hat{\theta}$ are the unit basis vectors.

(a) Write the vector equation of motion for a particle in an isotropic central force in plane polar coordinates.

(b) Split this into two scalar equations (a radial equation and an axial equation).

(c) Show that the axial equation constrains the motion of the particle. Describe the consequences of the axial equation.

The radial equation can be used to derive a differential equation describing the orbit of the particle, $r(\theta)$. To do this, one must eliminate the time derivatives from the radial equation so that the only variables are plane polar coordinates (r,θ) and derivatives of r with respect to θ.

To do this, a useful strategy is to use the following change of variable:

$$r = \frac{1}{u}$$

(d) Show that $\dot{r} = -h\dfrac{du}{d\theta}$ where h is the angular momentum per unit mass of the particle.

(e) Hence (or otherwise), derive an expression for $\ddot{r}$ in terms of h, u and its derivatives with respect to θ.

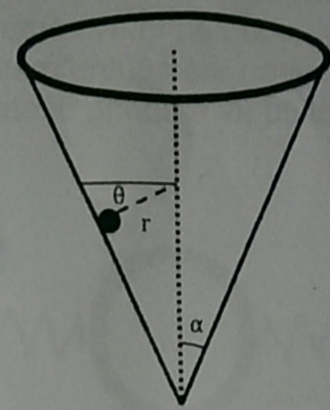
(f) Using these results (or otherwise), derive a differential equation for the orbit of a single particle of mass, m, in an isotropic central force field in terms of m, h and the variables u and θ.

(g) Show how this equation simplifies when the central force is proportional to the inverse square of the radius.

$$\frac{d^2u}{d\theta^2} + u + \frac{h}{mh} = 0$$

Question 3

PART A



A particle is confined to slide on the inside surface of a frictionless cone. The cone is fixed to a table so that its axis is perpendicular to the table. The cone half angle is α. The coordinates of the particle are given by r, the distance from the axis of the cone and θ the angle around the axis. Gravity acts on the particle.

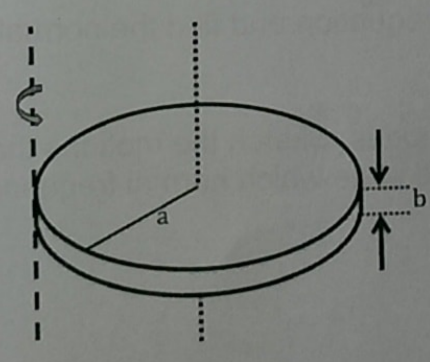
(a) Derive an expression for the total kinetic energy of this system. (Hint: you must consider motion around the cone as well as motion perpendicular to this up the side of the cone.)

(b) Derive an expression for the Lagrangian of this system.

(c) Use Lagrange's equations to find the equations of motion for this system.

(d) Show that the angular momentum of the particle is conserved.

PART B

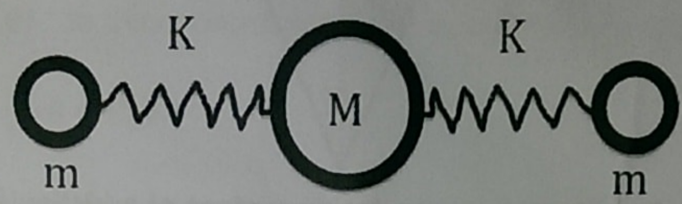


A solid disc of mass, M, radius, a, and thickness, b, rotates about an axis that touches its rim and is parallel to its symmetry axis.

(e) Derive an expression for the moment of inertia of the disc about its rotation axis.

Question 4

A linear triatomic molecule can be modelled using classical mechanics as comprising a central mass, M, coupled to two smaller masses, m (one on each side). The coupling between the central mass and each of the two smaller masses can be modelled as a spring of stiffness K.



For simplicity, only consider motion along the axis of this molecule, i.e. one dimensional motion.

(a) How many degrees of freedom does this system have assuming it is confined to motion along its axis (i.e. one dimension)?

(b) Select a set of generalised coordinates to describe the state of this model for a linear triatomic molecule. Show your coordinates on a sketch of the molecule.

(c) Write expressions for the kinetic energy T and the potential energy V for this molecule and hence write an expression for the Lagrangian.

(d) Derive the equations of motion for this molecule.

(e) Derive the secular equation for the system.

(f) Solve the secular equation and find the normal frequencies of the system.

(g) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.