THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – APRIL 2014

PHYS2010 – MECHANICS

PHYS2120 – MECHANICS & COMPUTATIONAL PHYSICS (Mechanics Paper)

Time allowed – 50 minutes

Total number of questions – 3

Answer ALL THREE questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{c}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 e^{i\omega t}$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$V_{eff}(r) = \frac{L^2}{2mr^2} + V(r)$$
$$\theta = \pm\int \frac{\left(L/r^2\right)dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$
$$t = \pm m\int \frac{dr}{\sqrt{2m\left[E - V_{eff}(r)\right]}}$$

Lagrangian

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

Question 1

The damped harmonic oscillator is described by the equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(i) Describe the types of motion that will be displayed by this oscillator and how they depend on the three parameters in the equation of motion.

(ii) When this system oscillates, what is the frequency of oscillation?

(iii) How does this compare with the comparable undamped simple harmonic oscillator (i.e. set $c = 0$)?

The above damped oscillator is driven by a harmonic driving force given by:

$$F = F_o e^{i\omega t}$$

(iv) Give an equation for the amplitude of the driven oscillator as a function of the driving frequency (you do **not** need to derive this equation!).

(v) Sketch this amplitude as a function of driving frequency for the oscillator under (a) weak damping and (b) very strong damping.

(vi) What condition on the oscillator parameters will determine whether this system shows resonance?

(vii) What is the resonant frequency of the driven damped harmonic oscillator and how does this depend on the damped oscillator parameters?

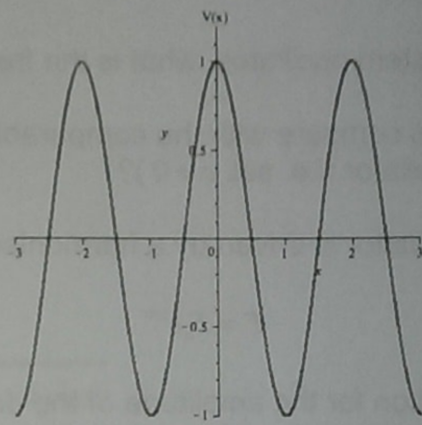
(viii) Sketch the relationship between the phase of the oscillating particle motion and the phase of the driving force as a function of frequency. Label all significant points. Include an example of weak damping and very strong damping.

Question 2

A particle can move in one dimension under the influence of the potential function:

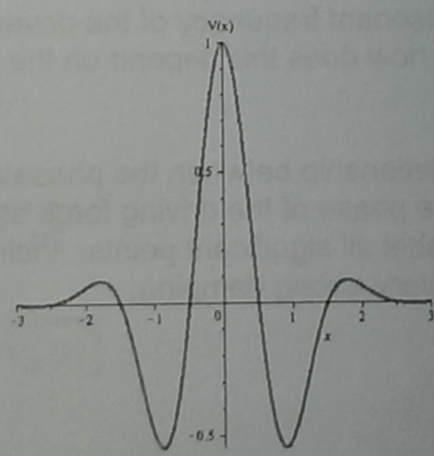
$$V(x) = \cos(\pi \cdot x)$$

which is shown in the graph:



(i) Plot the velocity phase space portrait of this function. On your plot mark: stable equilibrium points, unstable equilibrium points, the separatrix, regions of bounded and unbounded motion.

The potential function is modified so as to suppress the oscillations, leaving only the central maximum, two subsidiary maxima and two minima, as shown in the following diagram:



(ii) Plot the velocity phase space portrait of the modified potential function. On your plot mark: stable and unstable equilibrium points, separatrix(es).

(iii) Discuss the different types of motion described in this velocity phase space portrait for the modified potential function.

Question 3

In plane polar coordinates, the velocity is given by:

$$\mathbf{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$$

where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are the unit vectors.

A particle of mass m moves under the action of an isotropic central force:

$$\mathbf{f}(r) = f(r)\hat{\mathbf{r}}$$

(i) Write an expression for the Lagrangian of the particle in an isotropic central force using plane polar coordinates.

(ii) Use the Lagrangian to derive two equations of motion for this system.

The Lagrangian does not explicitly depend on the angular variable θ.

(iii) Show that this implies that the angular momentum of the particle is conserved in the presence of the central force.

(iv) Use the conservation of angular momentum to derive a radial equation of motion that depends only on r and its total time derivatives ($\dot{r}$ and/or $\ddot{r}$).

The isotropic central force is determined by the potential:

$$V(r) = \beta r^k$$

(v) Derive an expression for the central force produced by this potential.

(vi) Using the radial equation of motion derived in part (iv) (or otherwise), determine the radius of the circular orbit of the particle for a given angular momentum L.