THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

EXAMINATION – JUNE 2011

PHYS2010 – MECHANICS

Time allowed – 2 hours

Total number of questions – 4

Answer ALL FOUR questions.

The questions are of equal value.

This paper may be retained by the candidate.

Candidates may not bring their own calculators.

All answers must be in ink.

Except where they are expressly required, pencils may only be used for drawing, sketching or graphical work.

FORMULA SHEET

Damped Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = Ae^{qt}$$
$$q = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$
$$\gamma = \frac{b}{2m}$$
$$\omega_0^2 = \frac{k}{m}$$

Forced Harmonic Motion

$$m\ddot{x} + b\dot{x} + kx = 0$$
$$x = A\cos(\omega t - \varphi)$$
$$A = \frac{F_0}{m\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2\omega^2}}$$
$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$
$$\omega_r^2 = \omega_0^2 - 2\gamma^2$$
$$Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\gamma}$$

Central field

$$U_{eff}(r) = \frac{M^2}{2mr^2} + U(r)$$
$$\varphi = \pm \int \frac{\left(M/r^2\right)dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$
$$t = \pm m \int \frac{dr}{\sqrt{2m\left[E - U_{eff}(r)\right]}}$$

Lagrangian

$$L = T - U$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

Question 1

For the damped harmonic oscillator, the equation of motion can be written as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(a) explain all terms in this equation.

This results in three distinct types of solutions that represent three types of physical behaviour: overdamping, critical damping and underdamping.

(b) Explain each of these modes of physical behaviour. Use sketches to illustrate the motion of the particle in each case.

The above oscillator is driven by a harmonic driving force given by:

$$F(t) = F_o e^{i\omega t}$$

On solving the equations of motion for this driven damped harmonic oscillator, one finds that the amplitude, $A(\omega)$, as a function of frequency is given by:

$$A(\omega) = \frac{F_o / m}{\left[\left(\omega_o^2 - \omega^2\right)^2 + 4\gamma^2\omega^2\right]^{\frac{1}{2}}}$$

(c) Explain the meaning of all the symbols in the equation for $A(\omega)$.
(d) Derive an equation for the resonant frequency of the system.
(e) Sketch $A(\omega)$ as a function of ω for various values of γ. Be sure to plot and label curves where the behaviour of the system changes.

The phase shift in the driven damped harmonic oscillator is given by:

$$\tan(\phi(\omega)) = \frac{2\gamma\omega}{\omega_o^2 - \omega^2}$$

(f) Explain the meaning of this equation.
(g) Plot ϕ as a function of ω for several values of γ. Label all important parameters on your graph (common points, asymptotes, inflection points etc).

Question 2

A central force is given by:

$$f(r) = -\frac{k}{r^3}$$

where k is a constant and r is the distance from the centre. An object of mass m moves under the influence of this central force.

(a) Show that the potential producing this force is given by:

$$V(r) = -\frac{k}{2r^2}$$

(b) Derive an expression for the effective potential, U(r), of the particle, mass m, moving in this central field.

The form of the equation for the effective potential produces three distinct curves that depend on the values of the parameters: m, the particle mass; h (or l), the angular momentum per unit mass; and k, the force constant.

(c) Sketch the three possible effective potential curves for this central force as a function of radius. Label each curve with the conditions on m, h (or l) and k that are appropriate to the curve.

(d) What type of motion will be experienced by the particle in each of the three cases?

(e) Use Newton's second law (or Lagrange's equations) to show that the equation of motion for the particle reduces to:

$$\ddot{r} + \frac{1}{r^3}\left(\frac{k}{m} - h^2\right) = 0$$

(f) Hence show that the orbit equation for the particle is given by:

$$\frac{d^2u}{d\theta^2} - u\left(\frac{k}{mh^2} - 1\right) = 0$$

where u=1/r and θ is the angle in plane polar coordinates.

Question 3

A double pendulum consists of two masses m_1 and m_2 in the gravitational field g directed down. The mass m_1 is attached to a massless rod of length l_1 which hangs from a pivot attached to the ceiling, while m_2 is attached to another massless rod of length l_2 connected by a pivot with m_1.

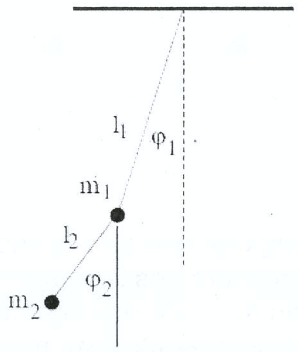

FIG. 1:

The dashed lines just show the vertical direction. The masses can move only in the plane of Figure, so the natural dynamical variables are angles φ_1 and φ_2.

(a) Derive Lagrangian of the pendulum in terms of φ_1 and φ_2.

(b) Derive equations of motion of the pendulum. Do not assume that the angles are small.

Question 4

Two equal masses m can move without friction. The masses are connected by a spring with elastic constant k. An identical spring connects the left mass to the wall.

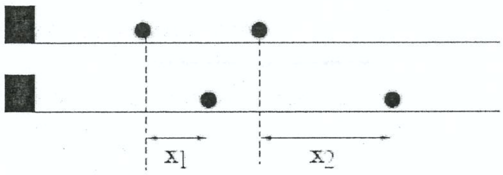

FIG. 2:

The masses can move only along one line as it is shown in Figure. The top part of the figure shows the equilibrium position (springs are un-stretched). The lower part shows a deviation from the equilibrium. So the coordinates x_1 and x_2 denote displacements of each mass from the corresponding equilibrium position.

(a) Write down the Lagrangian for this system in terms x_1 and x_2 and hence derive the equations of motion.

(b) Derive the secular equation for the system.

(c) Solve the secular equation and find the normal frequencies of the system.

(d) Find the normal modes. Sketch the motions that correspond to the normal modes and state which normal frequency corresponds to which mode.