THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MIDSESSION TEST – APRIL 2006

PHYS2010 – MECHANICS

Time allowed – 50 minutes

Total number of questions – 3

Answer ALL questions.

The questions are of not of equal value.

This paper may be retained by the candidate.

NO calculators are to be used for this paper.

All answers must be in ink.

Except whre they are expressly required, pencils may only be used for drawing, sketching or graphical work.

The following equations are supplied as an aid to memory.

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

Damped Harmonic Motion

If $m\ddot{x} + c\dot{x} + kx = 0$

then $x = Ae^{qt}$

where $q = -\gamma \pm (\gamma^2 - \omega_o{}^2)^{1/2}$

$\gamma = c / 2m$

$\omega_o{}^2 = k / m$

$\omega_d{}^2 = \omega_o{}^2 - \gamma^2$

Forced Harmonic Motion

If $m\ddot{x} + c\dot{x} + kx = F_o \cos \omega t$

then $x = A \cos(\omega t - \phi)$

where $$A = \frac{F_o}{\left[m^2\left(\omega_o{}^2 - \omega^2\right)^2 + c^2\omega^2\right]^{1/2}} = \frac{F_o}{m\left[\left(\omega_o{}^2 - \omega^2\right)^2 + 4\gamma^2\omega^2\right]^{1/2}}$$

and $$\tan \phi = \frac{c\omega}{m\left(\omega_o{}^2 - \omega^2\right)} = \frac{2\gamma\omega}{\left(\omega_o{}^2 - \omega^2\right)}$$

resonance $\omega_r{}^2 = \omega_o{}^2 - 2\gamma^2$

$$Q = \frac{\omega_d}{2\gamma}$$

Central Forces

Polar Coords

$\mathbf{r} = (r, \theta)$

$\mathbf{v} = (\dot{r}, r\dot{\theta})$

$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2, 2\dot{r}\dot{\theta} + r\ddot{\theta})$

$$u = \frac{1}{r} \rightarrow \frac{d^2u}{d\theta^2} + u = -\frac{1}{mh^2u^2} f(u^{-1})$$

$$e = (r_a - r_p) / (r_a + r_p)$$

$$e = \frac{mh^2}{kr_p} - 1$$

h = angular momentum / unit mass

Apsidal angle
$$\psi = \pi\left(3 + a\,\frac{f'(a)}{f(a)}\right)^{-1/2}$$

Stability
$$f(a) + \frac{a}{3}\,f'(a) < 0$$

Inverse Square Law Orbits
$$V = -\frac{k}{r}$$

Gravitation
$$k = GMm$$

$$\dot{\theta} = hu^2$$
$$f(r) = -k/r = -\frac{GMm}{r^2}$$
$$e^2 = \frac{a^2 - b^2}{a^2}$$
$$\frac{1+e}{1-e} = \frac{r_a}{r_p}$$
$$\tau = \frac{2\pi}{\sqrt{GM}}\,a^{3/2}$$
$$G = 6.67\times10^{-11}\ N\ m^2kg^{-2}$$
$$M_{sun} = 2\times10^{30}\ kg$$
$$M_{earth} = 6\times10^{24}\ kg$$
$$R_{earth} = 6400\ km$$

Lagrange's Equations
$$L = T - V$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0$$

Generalized momenta
$$p_j = \frac{\partial L}{\partial \dot{q}_j}$$
$$H = \sum p_i\dot{q}_i - L$$
$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Question 1 (7 marks)

A simple harmonic oscillator consists of a mass m attached to a massless spring, stiffness k.

(a) Using either Newton's laws or the Lagrangian (or otherwise), determine the equation of motion for this system.

(b) Solve the equation of motion for the simple (no damping) harmonic oscillator.

(c) Sketch x(t) for the simple harmonic oscillator (assuming initial position x=0 with a velocity $+v_o$). Mark all parameters on your graph and relate them to the initial conditions plus k and m.

(d) Sketch the velocity phase space portrait for this simple harmonic oscillator system.

(e) Determine the Lagrangian for this system.

A constant force, F_o is applied to the mass (parallel to the direction of the spring force).

(f) How does this constant force affect the motion of the oscillator?

(g) Determine the equation of motion for the oscillator in the presence of the constant force.

Question 2 (8 marks)

A particle of mass m moves under the influence of a central force F(r). Central forces are conservative and hence there exists a potential energy V(r) that corresponds to the force F(r).

(a) Write an expression for the kinetic energy of this particle in terms of the cylindrical polar coordinates (r, θ).

(b) Hence, write an expression for the total energy, E, of the system in terms of r, θ and their total time derivatives.

The total energy equation will contain terms in the total time derivative of θ.

(c) Using the conservation of angular momentum (or otherwise), eliminate the terms in the total time derivative of θ from the energy equation.

(d) Using the conservation of energy (or otherwise), derive an expression for the radial velocity, dr/dt, in terms of the total energy, the potential energy, the mass, m, the angular momentum and the variable, r.

The energy equation derived in part (c) contains only a single variable, r, and its total time derivative. Thus it describes the motion of a one dimensional system.

(e) Show that this equation can be analysed in terms of a radial kinetic energy and an effective potential.

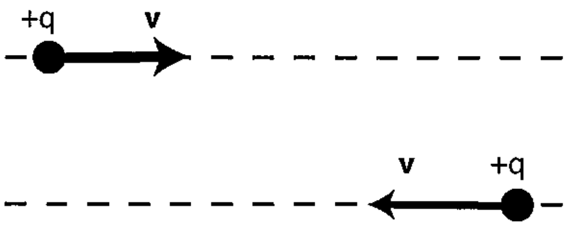
(f) Explain the terms in the effective potential.

(g) Sketch the effective potential (for a typical central force – you choose!) as a function of radius.

(h) Describe the possible motions of your system for different values of the total energy of the system.

Question 3 (5 marks)

Two charged particles travel along parallel paths in opposite directions at the same speed. Each particle has a positive charge +q. Ignore electrostatic forces.



(a) On a diagram, show the vectors representing the magnetic force produced on each particle by the magnetic interaction with the other particle. (Hint: use the Lorentz force:

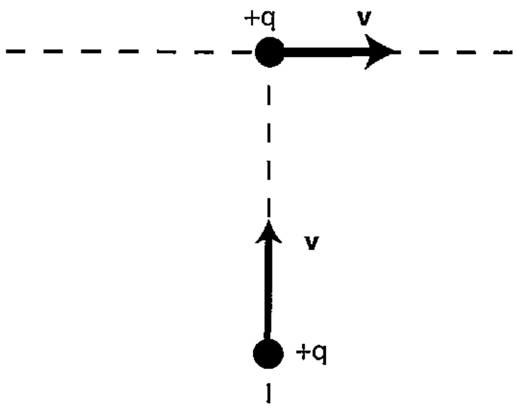
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

but assume that E=0).

(b) Discuss your result in terms of Newton's third law "for every action there is an equal and opposite reaction".

Part B

In a similar problem to *Part A*, two charges (each of charge +q) travel with the same speed v on perpendicular paths. Again ignore electrostatic forces.



(c) At the point where the velocity of one charge is exactly pointing at the other charge, sketch a diagram marking the force vectors on each particle due to the magnetic interaction with the other particle.

(d) Discuss your result in terms of Newton's third law of motion.