THE UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF PHYSICS

MID-SESSION TEST –APRIL 2003

PHYS2010 – MECHANICS

Time allowed – 1 hour

Total number of questions – 2

Attempt **ALL** questions

The questions are of **EQUAL** value

This paper may be retained by the candidate

The following equations are supplied as an aid to memory.

Damped Harmonic Motion

If $m\ddot{x} + b\dot{x} + kx = 0$

then $x = Ae^{qt}$

where $q = -\gamma \pm \left(\gamma^2 - \omega_o^2\right)^{1/2}$

$\gamma = b/2m$

$\omega_o^2 = k/m$

$\omega_d^2 = \omega_o^2 - \gamma^2$

Forced Harmonic Motion

If $m\ddot{x} + b\dot{x} + kx = F_o \cos \omega t$

then $x = A\cos(\omega t - \phi)$

where $A = \dfrac{F_o}{\left[m^2\left(\omega_o^2 - \omega^2\right)^2 + b^2\omega^2\right]^{1/2}} = \dfrac{F_o}{m\left[\left(\omega_o^2 - \omega^2\right)^2 + 4\gamma^2\omega^2\right]^{1/2}}$

and $\tan\phi = \dfrac{b\omega}{m\left(\omega_o^2 - \omega^2\right)} = \dfrac{2\gamma\omega}{\left(\omega_o^2 - \omega^2\right)}$

resonance $\omega_r^2 = \omega_o^2 - 2\gamma^2$

$$Q = \frac{\omega_d}{2\gamma}$$

Central Forces

Polar Coords

$\mathbf{r} = (\mathrm{r}, \theta)$

$\mathrm{v} = \left(\dot{\mathrm{r}}, \mathrm{r}\dot{\theta}\right)$

$\mathbf{a} = \left(\ddot{\mathrm{r}} - \mathrm{r}\dot{\theta}^2, 2\dot{\mathrm{r}}\dot{\theta} + \mathrm{r}\ddot{\theta}\right)$

$u = \dfrac{1}{r} \rightarrow \dfrac{d^2u}{d\theta^2} + u = -\dfrac{1}{m\ell^2u^2} f\left(u^{-1}\right)$

$\mathrm{r}_o = \dfrac{m\ell^2}{k(1+e)}$

$\mathrm{r}_1 = \mathrm{r}_o \dfrac{1+e}{1-e}$

$\ell =$ angular momentum/unit mass

Apsidal angle $\psi = \pi\left(3 + a\frac{f'(a)}{f(a)}\right)^{-\frac{1}{2}}$

Stability $\quad f(a) + \frac{a}{3} f'(a) < 0$

Inverse Square Law Orbits

$$V = -\frac{k}{r}$$

Gravitation

$$k = GMm$$

$$\dot{\theta} = lu^2$$

$$f(r) = -k/r = -\frac{GMm}{r^2}$$

$$e = \left(1 + \frac{2Em\ell^2}{k^2}\right)^{\frac{1}{2}}$$

$$\tau = \frac{2\pi}{\sqrt{GM}} a^{3/2}$$

$$G = 6.67 \text{ x } 10^{-11}\ N\ m^2\ kg^{-2}$$

$$M_{sun} = 2 \times 10^{30}\ kg$$

$$M_{earth} = 6 \text{ x } 10^{24}\ kg$$

$$R_{earth} = 6400\ km$$

Lagrange's Equations

$$L = T - V$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0$$

Generalized momenta

$$p_j = \frac{\partial L}{\partial \dot{q}_j}$$

$$H = \sum p_i \dot{q}_i - L$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i},\ \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

Question 1. [20 marks]

a) Consider a damped harmonic oscillator (unforced). Write down the general form of the displacement x of the oscillator as a function of time for the three cases when the motion is underdamped, critically damped, or overdamped.

A man of mass 100 kg. riding a light unicycle acquires an upwards velocity of 4 m.s.$^{-1}$ upon riding over a bump. The unicycle is fitted with a spring of stiffness $k = 1600$ N.m.$^{-1}$ and a shock absorber which provides linear damping with damping constant $b = 800$ N. m.^{-1}s.

b) Write down the equation of motion for the rider's vertical displacement from his equilibrium position after the bump. Is the motion underdamped, overdamped, or critically damped?

c) Determine the solution corresponding to the initial conditions above, assuming the initial displacement is zero. Sketch a graph of the displacement versus time.

V d) What is the maximum vertical height reached by the rider?

1.026 m

Question 2. [20 marks]

According to Yukawa's theory of nuclear forces, the attractive force between a neutron and a proton has the potential

$$V(r) = -\frac{Ke^{-\alpha r}}{r} \tag{1}$$

where K, α are positive constants.

a) Sketch the potential as a function of r. How does it compare with the potential of the inverse square force?

b) Find the corresponding force.

c) Discuss the types of motion of a particle of mass m in such a potential for different regimes of the total energy.

d) For a particle of mass m moving in a circle of radius a in the potential above, find:

   i) the total energy E;

   ii) the angular momentum per unit mass l;

   iii) the period of the circular motion.

c) Would you expect the motion of a bound particle to describe an ellipse, in general? Justify your answer.